Random and Programmed Pulse-Width Modulation Techniques for DC-DC Converters *

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Abstract

Programmed pulsed-width modulation (PWM) and randomized PWM techniques have been developed by numerous authors to null or suppress certain harmonic components of switching noise generated by switched power electronic circuits. This paper develops an approach for minimizing worst case spectral components of certain circuit waveforms in DC-DC converters. The approach is based on a programmed PWM method for harmonic reduction.

1 Introduction

This paper investigates methods for reducing unwanted spectral components of circuit waveforms in PWM power conversion circuits. Figure 1 shows a portion of a 50% duty cycle square wave and its spectral components. In many applications, the concentration of harmonic energy at discrete components is undesirable. For instance, it may be necessary to reduce an annoying audible tone, or an electromagnetic interference (EMI) specification may be satisfied by reducing peak harmonic components, or one might be able to reduce the size and weight of filter elements by reducing peak spectral components.

The main approaches considered in this paper rely on the fact that it is possible to operate a PWM type power circuit with a time-varying switching frequency provided the average duty cycle is not disturbed. In particular, we demonstrate how the switching instants can be perturbed in both programmed and random ways so that the power spectral density of a given circuit waveform is effectively smoothed out. Section 2 of the paper reviews the main techniques considered in the literature. Section 3 illustrates the application of these methods in the context of a DC-DC converter. That section compares the spectral densities obtained with the programmed and randomized PWM techniques and speculates on the possibility of combining the two approaches.

2 Review of Existing Techniques for Harmonic Control

The first method of harmonic control to be discussed can been termed programmed pulse-width modulation. In programmed PWM, a precomputed set of turn-on and turn-off times (or angles) is stored in a memory or look-up table, and then accessed periodically by a control circuit. This

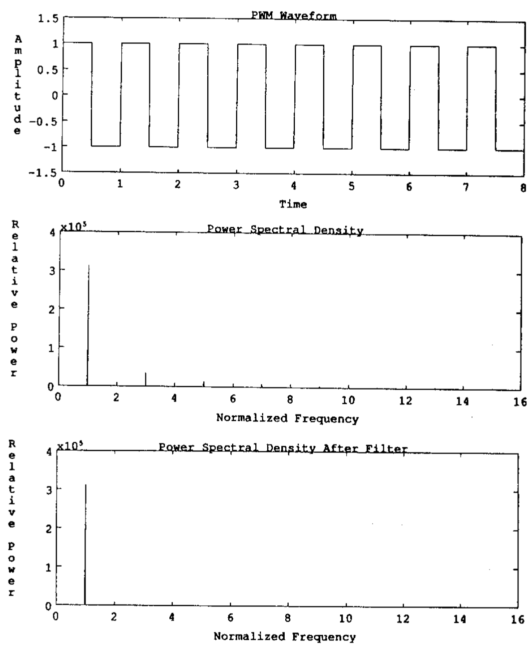

Figure 1: 50% Duty Cycle Square Wave, Associated Power Spectral Density, and Power Spectral Density Filtered by Two-Pole Roll Off

method can be used to generate steady state waveforms and therefore has found applications in inverters and dc-dc converters. Some of the earliest work in harmonic control is due to Turnbull [1]. In the context of an inverter, Turnbull calculated the necessary turn-on and turn-off angles to generate a modified square-wave that had no third or fifth harmonic components. Waveform symmetry properties, namely quarter-cycle even symmetry and half-cycle odd symmetry, were maintained to assure that no even harmonics were generated. Reference [1] also noted that certain three-phase connections blocked triple-n (i.e. third, sixth, etc.) harmonic components, and that it was advantageous in these three-phase connections to select the switching angles to null the fifth and seventh harmonics. The later work of Patel and Hoft [2,3] generalized the results of Turnbull by allowing, in principle, an arbitrary number of turn-on and turn-off transitions per cycle. The waveforms in [2,3] maintained the necessary quarter- and half-cycle symmetries to avoid the introduction of even harmonics. Reference [2] ap-

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plied a Newton-type numerical algorithm to determine the switching angles that resulted in a prescribed set of harmonics being nulled. Reference [3] extended the method of [2] to specify the amplitude of the fundamental as well as nulling a given set of harmonic components. The paper [5] of Goodarzi and Hoft took a similar approach to the harmonic control problem, but minimized a weighted square sum of certain harmonic components with the constraint that the amplitude of the fundamental assumed a constant value. This method also relied on a Newton-type numerical scheme for determining the optimal switching angles. A summary of the programmed PWM methods is given in the paper [4] of Enjeti et al.

A second approach to harmonic control involves randomized switching, and has been quite successfully applied by a number of researchers. It is of interest that with the randomized type switching, it is not necessary to precompute switching sequences, and it is therefore possible to apply the method to generate an arbitrary waveform. One example of this is the recent work of Tanaka et al. [6]. The focus in [6] was not on the lower harmonics of the fundamental switching frequency, but on much higher frequency noise generated by the turn-on and turn-off process in a given DC-DC converter. This work relied on the spectral characterization of the noise generated by each type of transition (turn-on or turn-off), as studied in the authors' earlier papers [7,8]. Using characterizations of the relevant transitions as step responses, the authors were able to deduce the form of the spectrum of a periodic sequence of turn-on and turn-off transitions. Reference [6] then added a random timing component to the switching sequence in an attempt to spread high frequency harmonic components. The random timing component introduced in [6] evidently corresponds to a frequency (or phase) modulation with white noise. The experimental results reported in [6] indicate that high frequency spectral components are reduced in magnitude by a factor of approximately two or three via the spectral spreading.

A very effective use of randomized switching has been reported in [9]. Reference [9] used a scheme very similar to that of [6] to reduce acoustical and electrical noise harmonics in a PWM induction motor drive. The main difference in [9] with respect to [6] is the use of bandlimited modulation. In particular, reference [9] relied on Carson's rule for the estimation of spectral bandwidths in wideband frequency modulation. The result was an effective shaping of the spectrum to reduce harmonic peaks. A reduction of the peak spectral components by a factor of approximately five was obtained. The paper [9] noted that the randomized switching could be implemented by generating the switch times in an on-line manner using a random number generator and shaping filter or by storing a long (a few seconds in length) sequence of switching periods.

Another implementation of randomized switching is in the paper of Trzynadlowski [10] where the ramp function used to implement PWM is replaced by a white sequence of uniformly distributed random variables. One apparent drawback of the scheme of [10] is the necessity of increasing the average switching frequency by a large factor.

3 Programmed and Randomized Switching Sequences for DC-DC converters

In order to study the application of programmed and randomized switching operation for a power converter, we shall

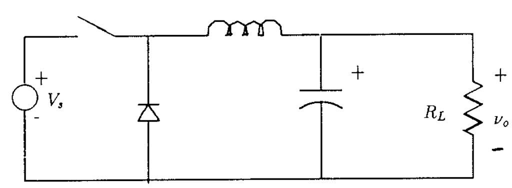

Figure 2: Buck Comverter

focus on the example buck converter of Figure 2. The example here is designed to operate at 50% nominal duty cycle with $v_o = V_s/2$. As previously discussed, the steady state switch waveform of Figure 1 results in relatively large harmonic components at discrete frequencies. The aim here is to compare how well programmed PWM, randomized switching, and a combination of these methods can perform in reducing worst case spectral components.

3.1 Programmed PWM: Min-Max Harmonic Magnitudes

The first approach to be considered is the programmed PWM method. For operation at the nominal switching frequency $f_s = 1/T$, this method can be implemented by selecting N subperiods $T_1, T_2, \ldots, T_n$ satisfying the constraints

$$T_1 + \ldots T_n = NT \tag{1}$$

and

$$T_k > 0 \tag{2}$$

for $k = 1, \ldots, N$. As discussed above, some of the previous approaches in programmed PWM sought to null a set of harmonic components or to minimize a weighted square sum of a set of the harmonic magnitudes. Here, we shall seek min-max type solutions. The desired solutions will minimize the magnitude of the largest non-DC spectral component, possibly with some appropriate weighting. We note that subharmonics will be introduced at the frequencies $1/NT, 2/NT, \ldots, (N-1)/NT$. The idea here is that energy from the fundamental at frequency $1/T$ can be spread among the subharmonics to reduce peak spectral energy. Such a solution should be satisfactory if all subharmonics remain above the audible range. For a PWM waveform constructed from N subperiods as described here, the magnitudes of the first N harmonics are given by

$$\alpha_n = \sum_{k=1}^{N} \frac{1}{N\pi} e^{\frac{-j2\pi n}{N}(p_1+p_2+\ldots+p_{k-1}+p_k d/2)}\, sin(\frac{n\pi p_k d}{N}) \tag{3}$$

where d is the duty ratio, and $p_k = T_k/T$ is the normalized length of the k-th subperiod.

Our approach for finding a min-max solution is based on the somewhat heuristic condition that the (weighted) magnitudes of the first N components should be equal. This condition can be justified by the idea that there are nominally $N-1$ free parameters due to the equality constraint (1) and so $N-1$ additional equality constraints may be satisfied. This is precisely the number of constraints imposed by the requirement that the first N (weighted) harmonic component magnitudes are equal. Note that if any one of the first N harmonic components has a magnitude smaller than the other $N-1$ components, it is possible to perturb the $N-1$ free parameters to decrease each of the $N-1$ largest harmonic components provided the map from free parameters to the $N-1$ largest harmonic magnitudes has a nonsingular Jacobian. With the condition stated here, the problem is reduced to finding a set of T_k (or p_k) satisfying the constraints (1) and (2) such that

$$w_1\alpha_1 = w_2\alpha_2 = \ldots = w_N\alpha_N \tag{4}$$

where the w_k are positive weights. One situation where the above condition may fail to correspond to a min-max solution is where higher harmonic components attain greater magnitudes than the lowest N components. There are still other other scenarios where the condition (4) does not correspond to a min-max solution. However, we shall see in our application below that this condition does lead to min-max solutions in certain problems.

An iterative algorithm for finding a solution $p_1, \ldots, p_N$ that satisfies the constraints (4) and (1) is outlined below.

1. Select an initial target point $\overline{\alpha}_n^0$ satisfying the condition (4).

2. Solve the least squares problem

$$\min_{p_k} \sum_{n=1}^{N} (\alpha_n - \overline{\alpha}_n^0)^2 \tag{5}$$

   This can be accomplished with a Newton type iteration. The result is a set of subperiods p_k^1 corresponding to the admissible harmonic magnitudes α_n^1. The point α_n^1 in the manifold of admissible magnitudes is nominally the projection of $\overline{\alpha}_n^0$ onto this manifold.

3. Update the target point to $\overline{\alpha}_n^1$ where $\overline{\alpha}_n^1$ is the point satisfying the constraint (4) with energy (l_2 norm) equal to the energy in α_n^1. Then, return to step 2 using the new target value. Increment all superscript indices as necessary.

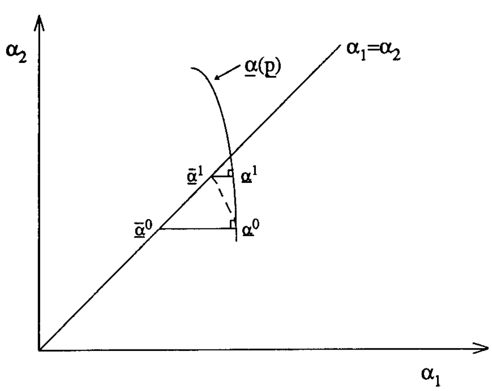

Figure 3: Illustration of Convergence of Algorithm

Figure 3 illustrates the progress of this algorithm for the case where the problem involves only two harmonic components. As suggested in the figure, the algorithm can converge quite rapidly.

Results The above algorithm was applied to determine an appropriate sequence of eight subperiods for the buck converter of Figure 2. The goal was to minimize the maximum spectral energy component (aside from dc) in the output

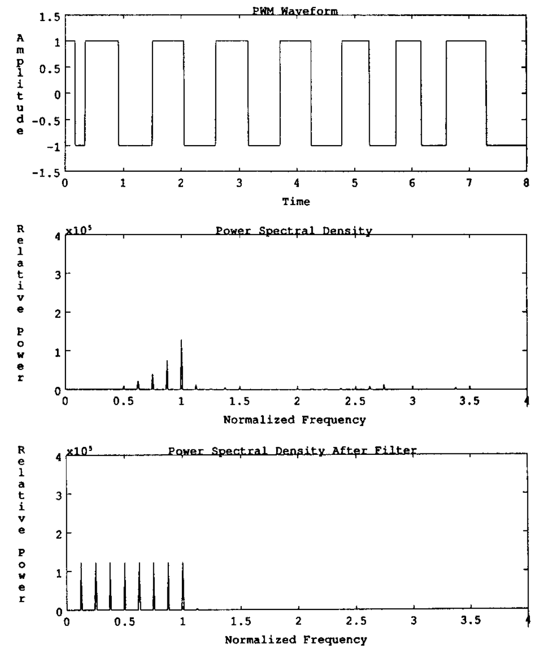

Figure 4: Optimal Eight Subperiod Switch Waveform, the Associated Power Spectral Density, and the Filtered Power Spectral Density

waveform of the buck converter. For the purposes here, the transfer function of the $L-C$ filter was approximated as a simple two-pole roll off, i.e. $1/s^2$. This approximation is based on the idea that the switching frequency is well above (by more than a factor of eight) the filter corner frequency. If this was not the case, the actual transfer function could be taken into account. With the two-pole roll off, it is necessary to weight the harmonics with weights of the form

$$w_k = (N/k)^2. \tag{6}$$

The result of the application of the algorithm outlined above with these weights is shown in Figure 4. It is evident that the method is effective in reducing peak spectral components, when compared with the square wave of Figure 1. By allowing longer sequences, it should be possible to further decrease the peak spectral density. However, there is an associated cost with the introduction of lower subharmonics. In principle, it is possible to appropriately weight these subharmonics so that they do not cause problems, but there are also the limitations of increased computational complexity and increased storage requirements for the implementation of the scheme.

In light of these limitations, we consider the application of randomized switching to attain the same objectives.

3.2 Randomized PWM

As discussed in Section 2, randomized switching techniques have been effectively applied to reduce peak spectral energy components in waveforms generated by PWM systems. The approach taken in [9] relied on colored noise and the use of Carson's rule for the estimation of effective bandwidths of certain spectral components. For purposes of comparison, we simulated such a random modulation scheme in

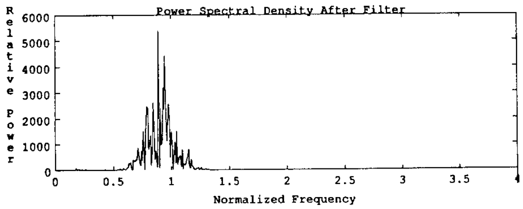

Figure 5: Power Spectral Density Estimate of Randomized PWM Waveform

the context of our example buck converter. For sinusoidal frequency modulation, the spectral bandwidth is given by approximately twice the maximum frequency deviation for the case of a large modulation index. That is, the maximum frequency deviation needs to be large compared to the bandwidth of the modulating signal. For this purpose, a low frequency noise process with bandwidth of approximately one-tenth the nominal switching frequency was used to modulate the switching frequency. The maximum frequency deviation was taken as approximately one-half the switching frequency. The result corresponding to the spectral power density of the converter output voltage is shown in Figure 5. As expected the bandwidth is approximately equal to the switching frequency, and the peak is significantly reduced with respect to the unmodulated squarewave of Figure 1.

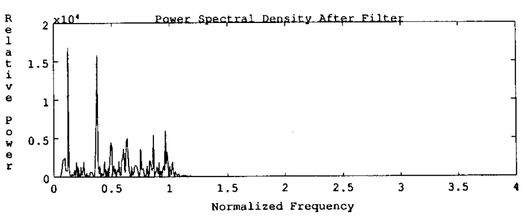

Figure 6: Spectral Density Estimate of Randomized Programmed PWM Waveform

A natural question that arises concerns the simultaneous application of programmed and randomized switching techniques. Can the spectral energy be most effectively spread out by combining these two approaches? To explore this possibility, the programmed switch waveform developed in the previous subsection was frequency modulated by the same low frequency noise process, but with a maximum frequency deviation of only one-fifth the switching frequency. A smaller frequency deviation was used since the spectral components of the programmed waveforms occur at lower frequencies and the spectral components are more closely spaced. The result is shown in Figure 6. As can be seen, the low frequency components appear to be accentuated. One possible cause for this is the spreading of the lowest harmonic component of the programmed PWM waveform to frequencies where the low-pass filter provides little attenuation. In order to flatten the power spectral density, it may be possible to precompute a different programmed PWM switching scheme that takes this effect into account. This remains as a task for future work.

4 Conclusion

This paper has investigated the application of programmed and randomized PWM techniques for the reduction of unwanted spectral components in power converter waveforms. As demonstrated, it is possible to precompute an optimal sequence of subperiods to minimize worst case spectral components. These spectral components can also be effectively reduced by the introduction of high index random frequency modulation. A combination of these two approaches may prove most effective in reducing peak spectral energy density, although this remains as a task for future study. More work is also needed to characterize optimal randomized frequency modulation techniques. A different approach to this problem could rely on the introduction of chaos to the PWM switching process.

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