Observers for Flux Estimation in Induction Machines

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Abstract—**Flux estimation in induction machines is examined from the viewpoint of observer theory. It is pointed out that estimators presently used in connection with schemes such as field-oriented control are typically real-time simulations of machine equations, without feedback of any corrective prediction error. We show that corrective feedback can be used to speed up convergence of the flux estimates, and note that it can also reduce the sensitivity of the estimates to parameter variations.**

I. Introduction

FIELD-ORIENTED control has (along with some variants) emerged as an important approach to the control of ac machines, and continues to be discussed and developed in the literature (see [1]–[3], [27], [28], and references therein). Rotor flux estimation from the terminal variables (stator voltage and current, and rotor speed) is a key step in the most popular implementations of such field-oriented control for induction machines. The estimation schemes in use for this purpose are typically real-time simulations of the dynamic equations governing rotor flux. The task of rotor flux estimation may also be expected to arise in other approaches to control and monitoring of induction machines.

A framework for understanding and extending flux estimation schemes in a unified way is provided by observer theory, which is briefly reviewed in Section II. We use this framework in Section III to assess the literature on flux estimation in induction machines, pointing out the constrained convergence rates of the real-time simulations typically used. Section IV begins by examining a real-time simulator based on the equations governing the rotor circuit, as this is a preferred estimation scheme for field-oriented control. We then show analytically, and verify by numerical simulations, that the modifications suggested by observer theory yield flux estimators that converge faster. Though present field-oriented control schemes demonstrate high performance, it is believed that there will be situations where improved convergence of the flux estimator itself will be called for. Section V outlines a similar development for an estimation scheme based on the stator circuit. Both these schemes correspond to reduced-order observers; a full-order observer is derived in Section VI. The use of the observer structure to achieve objectives other than faster convergence rate, such as reduced sensitivity to parameter variations, is also mentioned in these sections. Section VII discusses issues related to sampled-data implementation of observers. A preliminary report on the work presented in this paper appeared in the conference paper [4], while several details not shown in the present paper are given in the thesis [5].

Some pioneering work along the directions pursued in this paper can be found in papers of Bellini *et al.* [6]–[8] and, more recently, [9]. Our results, while similar in several respects (even though initially developed completely independently), are perhaps more transparent. One reason for this is that our development views an observer as a real-time simulation corrected by feedback of a prediction error term, and thereby permits the observer approach to be directly related to existing flux estimation schemes. There is also significant overlap between our work and the interesting and thorough study in the recent thesis (in German) of Zägelein [10]. Some of these connections to [6]–[10] are noted at appropriate points in the paper.

It should be kept in mind that the basic methodology of observer design is applicable to a much broader class of problems than the particular (and important) one addressed here, and similar issues may be expected to arise in other situations involving estimation and control for electrical machine systems. One of the objectives of this paper, therefore, is to provide a paradigm for treatments in other contexts.

II. Essentials of Observer Theory

Consider a system modeled by the state-space description

$$x'(t) = Ax(t) + Bu(t) + Gd(t) \tag{1}$$

where $x(t)$ is an n-dimensional *state* vector, $x'(t)$ is its component-wise derivative, $u(t)$ is an m-dimensional vector of *known inputs,* and $d(t)$ is a k-vector of *unknown inputs* representing external disturbances and parameter uncertainties. Suppose the *measured outputs* of the system are modeled by

$$y(t) = Cx(t) + Hd(t) \tag{2}$$

where $y(t)$ is a p-dimensional vector. Here A, B, C, G, and H are all constant matrices of appropriate dimensions.

Observer theory is aimed at providing a real-time estimate $\hat{x}(t)$ of the state $x(t)$ in the above model, using only the known signals $u(\cdot)$ and $y(\cdot)$. The theory is well developed in the

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context of linear systems (see [11], for example) and many results are known in the case of nonlinear systems as well (see [12], [13], for example). A straightforward approach to providing a state estimate for the model (1) when $d(t)$ is unknown is via a real-time simulation of (1) that ignores the term $Gd(t)$, namely

$$\hat{x}'(t) = A\hat{x}(t) + Bu(t). \tag{3}$$

However, an *observer* for the system modeled by (1) and (2) goes one step further, in that one corrects the above real-time simulation by use of the discrepancy between the actual outputs $y(t)$ of the system and the prediction $C\hat{x}(t)$ of these outputs that is obtained by ignoring the term $Hd(t)$ in (2). This results in the system of equations

$$\hat{x}'(t) = A\hat{x}(t) + Bu(t) + K[C\hat{x}(t) - y(t)]. \tag{4}$$

The term in brackets is called the *prediction error*, and the matrix K is termed the *observer gain.* When $K = 0$, one recovers the simple real-time simulation of (3). Given $u(\cdot)$ and $y(\cdot)$, the system (4) can be solved by integrating forward in real time from some specified initial condition $\hat{x}(0)$, using analog and/or digital methods (e.g., using a microprocessor), thereby providing a state estimate.

The effectiveness of the observer is assessed by examining the dynamics of the estimation error

$$e(t) = \hat{x}(t) - x(t). \tag{5}$$

It is easily seen from (1), (2), and (4) that

$$e'(t) = (A + KC)e(t) - (G + KH)d(t). \tag{6}$$

The initial condition for (6) is the initial estimation error $e(0)$, which (even if small) is invariably nonzero because of uncertainties regarding the initial state of (1).

Consider first the case where the disturbance/uncertainty term $d(t)$ can be taken to be 0. The behavior of (6) is then governed by the eigenvalues of $A + KC$. If we set $K = 0$, i.e., if the real-time simulation is *not* corrected by a prediction error term, then the error dynamics is governed by the eigenvalues of the matrix A, and is thus the same as that of the underlying system (1). For a sluggish or unstable system (1), this is typically unacceptable, because the estimate $\hat{x}(t)$ will then converge only sluggishly or not at all to $x(t)$. We might expect that the error dynamics can be modified to obtain faster convergence of the estimate if we appropriately pick some nonzero observer gain K. It turns out—see [11]—that, under a so-called observability condition on the pair of matrices A, C, an appropriate choice of K can place the eigenvalues of $A + KC$ arbitrarily (subject only to the requirement that complex eigenvalues are specified in conjugate pairs). The error dynamics can therefore, in principle, be modified arbitrarily from that of the simple real-time simulator.

The unavoidable presence of system disturbances and model uncertainties, i.e., the fact that $d(t)$ is nonzero, causes the error behavior to not be governed purely by the eigenvalues of $A + KC$, as is evident from (6). Thus, even if one makes the simplifying assumption that $d(t) = 0$ when carrying out the observer design, one would not attempt in practice to make the observer overly fast. A fast observer would require large values for the entries of K, and these large values would in turn cause large entries in $G + HK$, with a corresponding accentuation of the effects of nonzero $d(t)$. In fact, the primary objective of observer design often shifts altogether to that of obtaining estimates that are less sensitive to disturbances/uncertainties, rather than obtaining estimates that converge fast. For example, the most familiar version of the celebrated Kalman filter (see references in [11]) is precisely an observer in which the gain K is chosen to give minimum mean square estimation error when $d(t)$ is modeled as a white Gaussian noise process. Our results in this paper focus entirely on obtaining faster flux estimates in induction machines, but we shall refer to work by others that is aimed at obtaining less sensitive estimates. It is also possible to consider combining these approaches, trading off speed of convergence against lowered sensitivity.

The review of elementary observer theory given in this section is only intended to expose the underlying philosophy of using a real-time simulation that is corrected by a prediction error term. There are many variants of the above situation that could be addressed. For example, the measured outputs may be given by a more complicated form than (2), perhaps involving the inputs, their derivatives, and state derivatives. In this case, one may want to construct the observer with modified variables so as to avoid differentiation of signals. This will be the situation for the particular class of observers we derive in Sections IV and V, but we defer the treatment of this case to those sections, rather than attempting a general statement here. Similarly, for machines operating with variable speed, our models will turn out to be time-varying, so that the associated dynamics is no longer simply characterized by eigenvalues of the governing matrices. Again, we shall defer treatment to the appropriate sections below, rather than trying to develop the requisite results in any generality here.

III. Assessment of Existing Flux Estimation Schemes

The literature on flux estimation for electrical machines *rarely* makes a clear distinction (notationally or otherwise) between the state of the system being studied and the state of the estimator itself, thus obscuring the issue of the behavior of the estimation error. There are some notable exceptions, the most explicit treatments being [6]–[10], [14], [15], and, in a Kalman filter setting, [19]–[21]. The distinction between the underlying state and its estimate is also made in [16] and [17], where the focus is on the effects of parameter errors. A further striking fact is that most flux estimation schemes that we are aware of correspond essentially to real-time simulations that have *no correction term* derived from the prediction error. This applies to [14] and [15] as well, even though they use the word "observer" in their titles.

Schemes that *do* use a corrective prediction error include [1], [6]–[10], [16], [18]–[22]. As already mentioned in Section I, the closest in spirit to our work are [6]–[10], and these will be mentioned again later. The results in [1], [10], [16], and [18] are primarily concerned with reducing sensitivity to uncertainties in parameters (especially the rotor time constant, which varies substantially with operating temperature) or to errors in measurements (of speed, for example—see [10] and [18]). Of these latter four references, the schemes in [10] and [18] have the same essential form as the observers that are discussed later in this paper, though the estimator in [18] is not presented as an observer. The schemes in [1] and [16], on the other hand, generate their prediction error terms in a slightly more intricate way, and use these to correct (on a much slower time-scale than the flux dynamics) the value of the assumed rotor time constant in a real-time simulator; they may thus be considered slowly adapting real-time simulators, rather than observers in the sense of this paper. The papers [19]–[22] proceed via an extended Kalman filter or least squares approach and therefore automatically have a corrective prediction error term. We may also note that [23] mentions the use of corrections to the real-time simulator but does not elaborate on this at all.

The discussion following (6) suggests that the dynamics of the estimation error in all those schemes that are simply real-time estimators is governed by the rotor time constant. We shall exhibit this fact in more detail in Section IV, and show how to obtain faster error decay by the use of a corrective prediction error signal. An alternative estimation scheme based on the stator circuit is examined in Section V, and is found to have *no* error decay mechanism! Again, the use of a corrective prediction error signal serves to remedy the situation.

There are perhaps two main reasons for the neglect of error dynamics in existing flux estimation schemes. Firstly, field-oriented control has been found in practice to be satisfactorily robust and effective without faster flux estimation than that provided by present schemes. Despite this first reason, the issue needs to be exposed and studied, because there will undoubtedly be applications where error decay at a rate limited by, for example, the rotor time constant may not be at all satisfactory. Secondly, existing theoretical treatments of this error dynamics, such as [6]–[9], [14], [15], [19]–[22], are not easily penetrated. This fact gives us additional motivation to examine the question of error dynamics.

It has already been noted in Section I that our study here of observers for faster flux estimation can also serve as a paradigm for similar treatments in the context of other estimation problems in machine systems. There is a clear trend toward replacing sensors that are expensive, unreliable, or hard to install, with less costly measurement schemes and more sophisticated signal processing; see, for example, the discussions in [1], [20], [22], [24], [27]. One will thus inevitably have to confront the issue of estimation errors and their dynamics in other contexts.

IV. Observer Based on the Rotor Circuit

A. A Real-Time Simulator

Consider the idealized two-axis model of a squirrel-cage induction machine that is used throughout the literature on field-oriented control (see, for example [1]). The rotor flux dynamics in this model satisfies

$$\lambda_r' = [(-1/T_r)I + wJ]\lambda_r + (1/T_r)Mi_s \tag{7}$$

where λ_r and i_s are two-component vectors that constitute the two-axis representations (in stator-fixed coordinates) of rotor flux and stator current, respectively, w is the angular velocity of the rotor (we are assuming without loss of generality that there is only one pole pair), T_r is the rotor time constant, M is the mutual inductance between rotor and stator, and

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \tag{8}$$

Note that λ_r, i_s, and w can, and generally will, all be functions of time, though we shall not explicitly show the time argument in order to avoid notational clutter. It should also be mentioned that λ_r and i_s can also be taken as complex numbers rather than real two-component vectors, with I and J then being interpreted as 1 and $\sqrt{-1}$, respectively (see [25], [27]).

The model (7) motivates the following real-time simulation that constitutes a common estimation schemes for rotor flux

$$\hat{\lambda}_r' = [(-1/T_r)I + wJ]\hat{\lambda}_r + (1/T_r)Mi_s. \tag{9}$$

It is assumed that rotor speed and stator current are known quantities, so that the state-space model (9) is in principle simply implemented, with analog or digital hardware to perform the required real-time integration of (9).

The error in the flux estimate produced by (9) is

$$e = \hat{\lambda}_r - \lambda_r \tag{10}$$

and is governed by the following state-space equation, obtained by subtracting (7) from (9):

$$e' = [(-1/T_r)I + wJ]e. \tag{11}$$

As mentioned in Section III, this equation is rarely displayed or examined explicitly in the literature, with [8], [9], [15], and [16] being exceptions (and with [6], [7], [10] and [14] displaying the analogous equation for the full-order observer, see Section VI).

For a given speed waveform w, (11) is a linear system. However, since w is in general a time-varying function, the convergence properties of (11) cannot in general be studied by simply taking the eigenvalues of the matrix in brackets. When the speed w is constant, this matrix becomes constant. In this case, since the eigenvalues of the matrix are $-1/T_r \pm jw$ (which is most easily seen if one understands the isomorphism between matrices of the form $aI + bJ$ and the complex number $a + jb$, see [25]), the two scalar components of e display an oscillation at the frequency w (the constant rotor speed) that is damped with a time constant of T_r (the rotor time constant).

Significant insight into the case of general, time-varying w is also fairly easily obtained. Premultiplying both sides of (11) by $2e^*$, where the * denotes transposition (or conjugation, if the complex variable notation is being used), and using the facts that $2e^*e' = (e^*e)'$ and $e^*Je = 0$, we see that the squared magnitude of the error satisfies the equation

$$(e^*e)' = -2(e^*e)/T_r. \tag{12}$$

The magnitude of the error thus decays with the time constant of the rotor. This analysis is really the Lyapunov function analysis in [15], with e^*e as Lyapunov function.

B. Using Prediction Error

Suppose now that we wish to have an estimate that converges faster than the one above. The philosophy underlying observer theory naturally suggests that a corrective signal derived from a prediction error should be added to the estimator in (9). We are thereby directly led to examine the following companion equation to (7), which completes the idealized description of the electromagnetic (as opposed to mechanical) variables associated with the machine, and which relates stator voltage v_s (a two-component vector) to stator current and rotor flux

$$v_s = (M/L_r)\lambda_r' + (\sigma L_s) i_s' + R_s i_s \tag{13}$$

where $\sigma = 1 - M^2/(L_s L_r)$, the leakage parameter, with L_r, L_s, and R_s being the rotor and stator inductances and the stator resistance, respectively; note also that $T_r = L_r/R_r$, where R_r is the rotor resistance. If the stator voltage is measured, it can be compared with the stator voltage predicted on the basis of (13), but using $\hat{\lambda}_r'$ instead of λ_r'

$$\hat{v}_s = (M/L_r)\hat{\lambda}_r' + (\sigma L_s) i_s' + R_s i_s. \tag{14}$$

The resulting observer then takes the form

$$\hat{\lambda}_r' = [(-1/T_r)I + wJ]\hat{\lambda}_r + (1/T_r)M i_s + K(\hat{v}_s - v_s) \tag{15}$$

where v_s is measured and $\hat{v}_s$ is computed from (14). K is a 2×2 matrix of observer gains. This is essentially the same observer as that of [8], [9], though our approach to it is somewhat different. It is also closely related to the flux estimator in [18], which will be further discussed later in this section.

We shall shortly show how to avoid having to take derivatives in implementing (14), (15). First, however, a straightforward calculation shows that the system (11) that governed error dynamics of the real-time simulation is now replaced by

$$e' = [-(1/T_r)I + wJ]e + (M/L_r)Ke' \tag{16}$$

or

$$e' = [I - (M/L_r)K]^{-1}[-(1/T_r)I + wJ]e. \tag{17}$$

It is evident that different choices of the observer gain matrix K will lead to different error dynamics.

Suppose, for illustration, that we pick

$$K = kI \tag{18}$$

where k is a scalar observer gain parameter. If the rotor speed w is constant, then (17) is a time-invariant linear system, and the eigenvalues that govern it are seen from (17), (18) to be

$$(1 - kM/L_r)^{-1}(-1/T_r \pm jw). \tag{19}$$

Thus, the eigenvalues of the error dynamics are scaled up by the factor $(1 - kM/L_r)^{-1}$, i.e., the time constant that governs the error decay is scaled down from that of the conventional observer by this factor, while the frequency of oscillation in the error decay waveform is scaled up by the same factor. For the more general time-varying case, we proceed as earlier to find that (12) is replaced by

$$(e^*e)' = -2(1 - kM/L_r)^{-1}(1/T_r)(e^*e) \tag{20}$$

so that the error magnitude now decays with a time constant of

$$(1 - kM/L_r)T_r. \tag{21}$$

It is evident that k can be chosen to make this time constant considerably smaller than T_r. (The analysis of the dynamics of the closely related observer in [8], [9] is carried out by invoking Wazewski's inequality, which involves essentially the same ideas.)

We now consider how to avoid having to take derivatives in implementing (14), (15). First define the auxiliary variable

$$z = [I - K(M/L_r)]\hat{\lambda}_r - K(\sigma L_s) i_s. \tag{22}$$

Then z' is simply obtained by grouping together all the terms of (15) that contain derivatives (after substituting for $\hat{v}_s$ from (14)), so that

$$z' = [(-1/T_r)I + wJ]\hat{\lambda}_r + (1/T_r)M i_s + K(R_s i_s - v_s). \tag{23}$$

Now (22) shows that

$$\hat{\lambda}_r = [I - K(M/L_r)]^{-1}[z + K(\sigma L_s) i_s]. \tag{24}$$

We can substitute this expression for $\hat{\lambda}_r$ in (23) to get a state equation for the vector z that is driven only by v_s and i_s, not their derivatives. This can be solved forward from whatever initial condition $z(0)$ is imposed by the choice of $\hat{\lambda}_r(0)$ in (22). To then find $\hat{\lambda}_r$ from z, one simply uses (24). Thus, no differentiation of signals is needed to implement this observer.

C. Illustrative Simulation Results

Some results of numerical simulations of the above observer design (using the program SIMNON, see description in [26]) are shown in Fig. 1. The machine (whose parameters are given in the figure) was considered to be excited by a sinusoidal 60-Hz voltage, and had a rotor time constant of 0.18 s. The trace in (a) shows the speed waveform for operation around 377 rad/s. The trace in (b) is the actual rotor flux component along axis 1. The traces associated with axis 2, both here and for the numerical simulations in the rest of the paper, are very similar (being approximately phase-shifted versions of the axis 1 traces), and are not shown. The trace in (c) shows the rotor flux estimate produced by the conventional real-time simulation scheme; (d) shows the estimate produced by our improved scheme above, with the choice $K = (L_r/2M)I$ to give an error time constant of $T_r/2$, according to (20); (e) plots the error in the conventional estimate; and (f) shows the error in our improved scheme, where the magnitude of the error decays with time constant $T_r/2$, but oscillates at a frequency of $2w$. It is evident that our scheme converges significantly faster.

D. Extensions

The particular gain in (18) was chosen for ease of illustration. A more general gain, suggested to us by results in [8], is

Parameters of machine used in numerical simulations		
R_s = 0.3	L_s = 0.0553 H	M = 0.0533 H
R_r = 0.3	L_r = 0.0546 H	

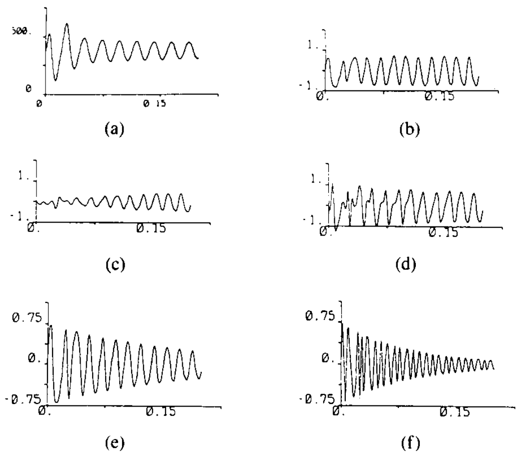

Fig. 1. (a) Speed. (b) Rotor flux on axis 1. (c) Conventional flux estimate (9). (d) Flux estimate of observer with corrective prediction error term (23), (24). (e) Error in conventional estimate (11). (f) Error in observer's estimate (17).

$$K = k_1 I + k_2 J. \tag{25}$$

If the rotor speed w is constant, the corresponding error system is again a time-invariant linear system. The governing eigenvalues are easily computed to be

$$-[g_1/T_r + g_2 w] \pm j[g_1 w - g_2/T_r] \tag{26a}$$

where

$$g_1 = (1 - Mk_1/L_r)/[(1 - Mk_1/L_r)^2 + (Mk_2/L_r)^2] \tag{26b}$$

$$g_2 = (Mk_2/L_r)/[(1 - Mk_1/L_r)^2 + (Mk_2/L_r)^2]. \tag{26c}$$

It is not hard to see from (26) that by proper choice of k_1 and k_2 one can place these eigenvalues at any specified pair of conjugate locations. It is in principle also possible to vary k_1 and k_2 as a function of (a slowly varying) operating speed, to control the variation of error dynamics with the operating point in some desired fashion, in accordance with (26); this sort of idea is explored (for a full-order observer, but similar ideas would apply here) in [6]. When w is not constant (and not slowly varying), a Lyapunov analysis of the kind used to obtain (12) and (20) can be carried out for the present case of the gain in (25), but we omit the details.

E. Effects of Parameter Uncertainty and Measurement Errors

It is important to examine the effects on the observer of errors in measured speed, in the current and voltage signals, and in the machine parameter values. We shall illustrate an approach to studying such effects by examining the case of uncertainty in the rotor resistance, a parameter that is known to vary considerably with the operating temperature [1], [16]. References to some of the literature on the effects of measurement and parameter errors will also be given.

Consider the implementation of the rotor flux observer (15) in the case where the assumed value of rotor resistance $\hat{R}_r$ is not equal to the actual value R_r. The observer then becomes

$$\hat{\lambda}_r' = [(-\hat{R}_r/L_r)I + wJ]\hat{\lambda}_r + (\hat{R}_r/L_r)Mi_s + K(\hat{v}_s - v_s). \tag{27}$$

By subtracting (7) from (27) and using the calculation

$$\begin{aligned}\hat{R}_r\hat{\lambda}_r - R_r\lambda_r &= \hat{R}_r\hat{\lambda}_r - \hat{R}_r\lambda_r + \hat{R}_r\lambda_r - R_r\lambda_r \\ &= \hat{R}_r e + (\hat{R}_r - R_r)\lambda_r \end{aligned} \tag{28}$$

we obtain the error dynamics

$$\begin{aligned} e' &= [I - (M/L_r)K]^{-1}[(-\hat{R}_r/L_r)I + wJ]e \\ &\quad + [I - (M/L_r)K]^{-1}(-\lambda_r + Mi_s)(\hat{R}_r - R_r)/L_r. \end{aligned} \tag{29}$$

It is thus seen that the error dynamics contains a driving term that is associated with the uncertainty in the rotor resistance. The error system (29) has the form of the error system (6) discussed in the section on observer theory. Note that the strength of the driving disturbance is dependent upon the state or operating point of the induction machine.

There are two general routes that may be followed in analyzing this error dynamics. If prior bounds on the machine state variables λ_r, i_s and on the uncertainty in the rotor resistance can be determined, then by picking an appropriate Lyapunov function (e.g., e^*e) the error e can be shown to be asymptotically confined to a certain bounded region about the origin. A second approach is to consider the behavior of (29) when the machine is in the sinusoidal steady state. The literature contains numerous studies using this latter approach.

Garces [16] displayed a similar error system to that of (29) with gain $K = 0$, but the focus in [16] (and in [1]) was to slowly update the assumed value $\hat{R}_r$ to reduce steady-state errors in the rotor flux estimate. Vagati and Villata [18] proposed an observer similar to the one here (though they did not label it an observer), with a particular gain that resulted in reduced sensitivity to proportional variations in R_r and R_s simultaneously and to measurement errors in the speed w. It is worth mentioning that this particular gain selection actually lead to a slower estimation error convergence rate than that of the real-time simulator. Bellini *et al.* [9] studied the effects of uncertainties in the stator and rotor resistances and the machine inductance parameters. In [9], steady-state estimation error results are tabulated for various operating speeds and for various parameter errors. The gains suggested in [9] for minimizing sensitivity to parameter uncertainties are generally different from $K = kI$. The thesis of Zägelein [10] contains the most comprehensive discussion of steady-state estimation error in conventional estimation schemes and in observers of the type discussed in this paper. It also demonstrates the improved performance attainable by use of an observer in field-oriented control of induction machines.

There are many other directions in which the above scheme needs to be further investigated and developed. The study of issues related to microprocessor implementation is of particular interest, see [9], [27], [28], and the comments in Section VII.

V. Observer Based on the Stator Circuit

There are many flux estimation schemes that may be considered, as evident from the review in [10]. One natural alternative to the schemes discussed in Section IV is based on rewriting (13) as

$$\lambda_r' = (L_r/M)(v_s - R_s i_s) - (\sigma L_r L_s/M) i_s' \tag{30}$$

which leads to the real-time simulation

$$\hat{\lambda}_r' = (L_r/M)(v_s - R_s i_s) - (\sigma L_r L_s/M) i_s'. \tag{31}$$

The limitation of this scheme that is typically quoted in the literature is the poor behavior at low rotor speeds. Note the additional fact, however, that the estimation error *remains constant,* because (if the scheme is actually implemented as above) the derivatives of actual and estimated flux are equal! There is no mechanism for decay of error induced by initial uncertainty or noise during operation.

One can now attempt to improve this estimator by feeding in a corrective prediction error term. In this case, it is (7) that we turn to for the prediction error term. The resulting observer then has the form

$$\hat{\lambda}_r' = (L_r/M)(v_s - R_s i_s) - (\sigma L_r L_s/M) i_s' + K(\hat{i}_s - i_s) \tag{32}$$

where v_s and i_s are measured, and $\hat{i}_s$ is obtained from (7) as

$$\hat{i}_s = (T_r/M)\{\hat{\lambda}_r' + [(-1/T_r)I + wJ]\hat{\lambda}_r\}. \tag{33}$$

The associated error system is then

$$e' = (T_r/M)K\{e' - [(-1/T_r)I + wJ]e\} \tag{34}$$

or

$$e' = -[(I - (T_r/M)K]^{-1}(T_r/M)K[(-1/T_r)I + wJ]e. \tag{35}$$

The parallel between this error model and the one in the previous section is evident. Once again, simple choices of K such as those in (18) or (25) will lead to error dynamics that is substantially different from that of the uncorrected system. Also, by use of appropriate auxiliary variables, it is again straightforward to implement the observer without use of differentiators.

The question that now arises is how one is to choose between the observer in this section and that in the previous section. The two are closely related, since they actually result from using the same sets of equations, and the error dynamics obtained by any particular choice of gain matrix in one scheme can typically be obtained by some appropriate choice of gain matrix in the other. However, it may be that one scheme is more easily implemented than the other in a given situation. For example, if one wanted the time constant of error decay to be T_r, with a gain of the form kI, then k would be 0 in the first scheme but infinite in the second. We have not carried out any detailed comparative study.

VI. Full-Order Observer

The observers presented above are actually examples of what are termed *reduced-order* observers, because they do not attempt to estimate *all* the state variables in our model of the system. A full-order observer would attempt to estimate, for example, i_s and w in addition to λ_r, because it is all of these variables (or equivalent combinations of them) that together make up the state variables in the model of our system. While i_s and w are generally easier to measure than λ_r, and are therefore often assumed (as in the previous two sections) to be available to us, the price one usually pays with the resulting reduced-order observers for λ_r is a strong sensitivity of the estimate $\hat{\lambda}_r$ to any noise in the measurements of i_s and w. For example, (24) shows that any noise in i_s appears (scaled but) unfiltered in $\hat{\lambda}_r$.

For this reason, even if i_s and w are available as measured signals, one might be motivated to construct a full-order observer. The estimate of i_s and w that the full-order observer produces will be filtered versions of the possibly noisy measurements of these variables, and therefore could (if the observer is properly designed) be preferable to the raw measurements for control purposes, e.g., in current- or speed-control loops. A proper analysis of this possibility, however, is hard. It would require introducing and studying the effect of noise models in the context of nonlinear models. References [19] and [20] represent beginnings in this direction. Our aim in this section is more modest.

We shall, in this section, consider a fourth-order observer that produces a filtered estimate of i_s along with one of λ_r, given measurements of i_s and w. We shall not attempt to produce a filtered estimate of w, as this would get us into the domain of nonlinear (as opposed to linear, time-varying) models. For this reason, our observer is actually "full order" only in the sense that it produces estimates of all the electromagnetic variables in the model. Assuming noise-free w, it is not so hard to obtain expressions for the effect of (certain types of) noise in v_s and i_s on the resulting estimates, but we omit this. It is considerably harder to assess the effect of noise in w, because of the way w enters the equations of the induction machine model, and we do not address this further here (but see [29]).

A. The Observer in [14]

The "observer" proposed in [14] is a fourth-order real-time simulation, without any corrective prediction error term. It generates estimates of both stator and rotor fluxes, using measured values of stator voltage and rotor speed. The present subsection reviews the analysis of [14] (displaying its key features in a much more direct way than [14]). The next subsection then examines the use of a corrective prediction error term, based on measurements of stator current.

The fourth-order system model under consideration is given by

$$\begin{bmatrix} \lambda_s' \\ \lambda_r' \end{bmatrix} = \left\{ -\boldsymbol{R}\boldsymbol{L}^{-1} + w \begin{bmatrix} 0 & 0 \\ 0 & J \end{bmatrix} \right\} \begin{bmatrix} \lambda_s \\ \lambda_r \end{bmatrix} + \begin{bmatrix} v_s \\ 0 \end{bmatrix} \tag{36}$$

where λ_s, λ_r, and v_s are two-component vectors representing stator flux, rotor flux, and stator voltage, respectively. The matrices $\boldsymbol{R}$ and $\boldsymbol{L}$ represent winding resistances and machine inductances, respectively,

$$\boldsymbol{R}=\begin{bmatrix} R_sI & 0 \\ 0 & R_rI \end{bmatrix}, \quad \boldsymbol{L}=\begin{bmatrix} L_sI & MI \\ MI & L_rI \end{bmatrix}. \tag{37}$$

If the observer is simply a real-time simulation of the above system model, as it is in [14], then the error model is easily seen to be given by

$$e'=\left\{-\boldsymbol{R}\boldsymbol{L}^{-1}+w\begin{bmatrix} 0 & 0 \\ 0 & J \end{bmatrix}\right\}e \tag{38}$$

where e is now the four-component vector denoting the error in the flux vector estimate.

If w is (nearly) constant, the dynamics of the error system is (approximately) governed by the eigenvalues of the system in (38). The special structure of (38), see [25], allows us to write the eigenvalues down explicitly, as the roots p_1, p_2 of the polynomial

$$p^2+[(1/T_s\sigma)+(1/T_r\sigma)-jw]p+(1-jwT_r)/(T_sT_r\sigma) \tag{39}$$

along with their complex conjugates p_1^*, p_2^*, where T_s is the stator time constant L_s/R_s.

Fig. 2 shows results of numerical simulations of the observer error transients for the same machine used for Fig. 1. Near 0 speed, the above eigenvalues are approximately -2.77 (twice) and -182.0 (twice). These values are reflected in the error transient associated with the rotor flux estimate on axis 1, shown in (a); the response is ultimately dominated by the larger time constant, which is 0.36 s ($=1/2.77$). At a speed of 377 rad/s, the eigenvalues are computed to be at $-93.0 \pm j354.0$ and $-91.7 \pm j22.7$. Again, these values are reflected in the rotor flux error waveform for axis 1, shown in (b); the time constant of the envelope is now 0.01 s ($=1/91.7$).

If w is not constant, a natural route to follow is Lyapunov analysis. Instead of examining the derivative of the squared length e^*e as before, we examine that of a *weighted* squared length, $e^*\boldsymbol{R}^{-1}e$. It is not hard to show from (38) that

$$\begin{aligned}(e^*\boldsymbol{R}^{-1}e)' &= -2(e^*\boldsymbol{L}^{-1}e)\\ &<0 \text{ for } e\neq 0\end{aligned} \tag{40}$$

where the inequality follows from the positive definiteness of $\boldsymbol{L}$ (or, less technically, from the fact that $L_s>0$ and $L_sL_r>M^2$). It can be shown from (40) that the weighted squared length of e decays at least as fast as an exponential of time constant

$$(\text{max eigenvalue of } L)/2\ (\min\{R_s,R_r\}). \tag{41}$$

Applying this bound to the constant speed case, and comparing with the exact eigenvalues given in the previous paragraph, shows that the bound in (41) tends to be conservative at higher speeds, but good at lower ones.

B. Using Prediction Error

We now consider improving the convergence of the real-time simulation above by use of a prediction error term

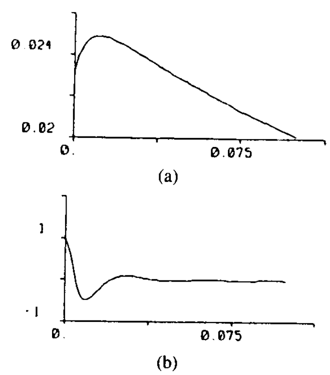

Fig. 2. Fourth-order "observer" without corrective prediction error term. Rotor flux estimation error on axis 1, (38). (a) at 0 rad/s and (b) at 377 rad/s.

derived from measurements of stator current. This approach has been taken earlier in [7], but our treatment is rather different.

It is convenient for our purpose to work with a model whose state variables are stator current and rotor flux. This is also a practical choice, since many control schemes need accurate estimates of these variables. The model is given by

$$\begin{bmatrix} i_s' \\ \lambda_r' \end{bmatrix}=\left[\begin{bmatrix} -aI & (M/bT_r)I \\ (M/T_r)I & (-1/T_r)I \end{bmatrix}+w\begin{bmatrix} 0 & -(M/b)J \\ 0 & J \end{bmatrix}\right]\begin{bmatrix} i_s \\ \lambda_r \end{bmatrix}+\begin{bmatrix} (L_r/b)I \\ 0 \end{bmatrix}v_s \tag{42}$$

where

$$a=(L_r^2R_s+M^2R_r)/(bL_r) \tag{43}$$

and

$$b=\sigma L_rL_s. \tag{44}$$

We propose an observer of the form

$$\begin{bmatrix} \hat{i}_s' \\ \hat{\lambda}_r' \end{bmatrix}=\left[\begin{bmatrix} -aI & (M/bT_r)I \\ (M/T_r)I & (-1/T_r)I \end{bmatrix}+w\begin{bmatrix} 0 & -(M/b)J \\ 0 & J \end{bmatrix}\right]\begin{bmatrix} \hat{i}_s \\ \hat{\lambda}_r \end{bmatrix}+\begin{bmatrix} (L_r/b)I \\ 0 \end{bmatrix}v_s+\begin{bmatrix} k_1I+k_2wJ \\ k_3I+k_4wJ \end{bmatrix}(\hat{i}_s-i_s) \tag{45}$$

where the k_i are scalars. Note the use of speed-dependent gains in (45). The resulting model for the observer error dynamics is then

$$e'=\left[\begin{bmatrix} (k_1-a)I & (M/bT_r)I \\ [k_3+(M/T_r)]I & (-1/T_r)I \end{bmatrix}+w\begin{bmatrix} k_2J & -(M/b)J \\ k_4J & J \end{bmatrix}\right]e. \tag{46}$$

Note that we can freely determine the scalar coefficients in the left-hand blocks of the two matrices in (46). If k_1 and k_3 are selected such that

$$k_1 - a = -k_2/T_r \tag{47a}$$

and

$$k_3 + (M/T_r) = -k_4/T_r \tag{47b}$$

the error dynamics becomes

$$e' = AQ(w)e \tag{48a}$$

where

$$A = \begin{bmatrix} k_2 I & (-M/b)I \\ k_4 I & I \end{bmatrix} \tag{48b}$$

and

$$Q(w) = \begin{bmatrix} (-1/T_r)I + wJ & 0 \\ 0 & (-1/T_r)I + wJ \end{bmatrix}. \tag{48c}$$

The freedom that we have in selecting k_2 and k_4 can be used to place the eigenvalues of A *in pairs* at arbitrary locations, as is easily verified by noting that the characteristic polynomial of A is

$$[p^2 - (1 + k_2)p + k_2 + k_4(M/b)]^2. \tag{49}$$

If the eigenvalues of A are p_1 (twice) and p_2 (twice), then the eigenvalues of $AQ(w)$ can be shown to be

$$[(-1/T_r) \pm jw]p_1 \text{ and } [(-1/T_r) \pm jw]p_2. \tag{50}$$

hence, if the machine speed w is (nearly) constant, the error dynamics is (approximately) governed by the eigenvalues in (50). It is therefore clear that in this case the observer error may be made to decay to zero exponentially fast, with a specified time constant. Fig. 3 shows the results of numerical simulations of the observer above, with $p_1 = 2$, $p_2 = 10$. The waveforms in (a) and (b) are obtained for a speed near 0, and correspond respectively to errors in the estimates of stator current and rotor flux on axis 1. Note that the error decay has time constant 0.09 s ($= T_r/p_1$). The waveforms in (c) and (d) correspond to these same two quantities, but are obtained for a speed near 377 rad/s. The results correlate well with the above analysis, in that the visible oscillation frequencies are 2 × 377 rad/s and 10 × 377 rad/s, while the envelope ultimately decays with time constant 0.09 s, as before.

If w is time-varying, the eigenvalues do not directly give information on dynamics, and it is natural to attempt a Lyapunov analysis. It is not hard to show that picking k_2 and k_4 to give real, positive p_1 and p_2 results in an exponentially bounded decay of the error, where the bound has time constant $T_r/\{\min\, p_1,\, p_2\}$. Thus, improvement in convergence rate over the observer in [14] can be guaranteed in this case as well. Further details may be found in [5].

VII. Sampled-Data Implementation of Observers

In this section we briefly examine some issues related to design of observers for sampled-data implementation. Such an

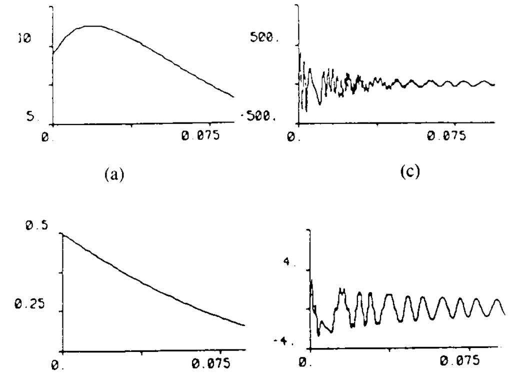

Fig. 3. Fourth-order observer with corrective prediction error term, (45). (a) Stator current estimation error on axis 1 at 0 rad/s. (b) Rotor flux estimation error at 0 rad/s. (c) Stator current estimation error at 377 rad/s. (d) Rotor flux estimation error at 377 rad/s.

implementation is natural in the context of microprocessor control. Consider first the linear, time-invariant, continuous-time system (1) with $d(t) = 0$, and suppose that its input $u(t)$ is constant over intervals of length T, starting at $t = 0$

$$u(t) = u(nT) \text{ for } nT \le t < (n+1)T, \quad n = 0, 1, \cdots. \tag{51}$$

It is then well known, see for example [26], that the evolution of the sampled state $x(nT)$ is described by the linear, time-invariant, *discrete-time* model

$$x(nT + T) = Fx(nT) + Gu(nT) \tag{52a}$$

where

$$F = \exp(AT) \text{ and } G = \int_0^T \exp(Aq)B\, dq \tag{52b}$$

with

$$\exp(Aq) = I + Aq + A^2q^2/2! + \cdots. \tag{52c}$$

The latter matrix is called the *matrix exponential.* Note that F and G are independent of the discrete time index n. The model in (52) is termed the sampled-data model corresponding to the continuous-time system (1).

If the underlying continuous-time system is time-varying, of the form

$$x'(t) = A(t)x(t) + B(t)u(t) \tag{53}$$

then the above procedure cannot be followed. However, if $A(t)$ and $B(t)$ can be taken as *piecewise constant* over intervals of length T, with values A_n and B_n in the nth interval, then one can still obtain a sampled model, except that it is now time-varying

$$x(nT + T) = F_n x(nT) + G_n u(nT) \tag{54a}$$

where

$$F_n = \exp(A_n T) \text{ and } G_n = \int_0^T \exp(A_n q)B_n\, dq. \tag{54b}$$

The major difficulty with this is the need to recompute the matrices in (54) at each time step to obtain the model that applies to that step. The main point we wish to make in this section is that the observers we have been considering so far have the feature that very little new computation is needed at each step.

To see this, return to the reduced-order observer (23), (24) of Section IV. If we assume that w is *constant*, and that v_s and i_s are *piecewise constant*, then a time-invariant sampled-data implementation of the form (52) is readily obtained by applying the above procedure. With the choice of gain K given in (18), the matrix exponential involved is

$$\exp\{(1-kM/L_r)^{-1}[(-1/T_r)I+wJ]T\}$$

$$=\exp(cT)\begin{bmatrix}\cos(dT) & -\sin(dT)\\ \sin(dT) & \cos(dT)\end{bmatrix} \quad (55a)$$

where

$$c=-(1-kM/L_r)^{-1}/T_r \quad (55b)$$

$$d=(1-kM/L_r)^{-1}w. \quad (55c)$$

The more interesting case occurs when w is *not* constant. Assuming that w is *piecewise constant,* taking the value w_n in the nth interval, one can obtain a time-varying sampled data model of the form (54). Note from (55) that the new model for each time step is easily computed when the new value of w is obtained. The reason for this is that the matrix exponential in this case is a simple function of w. Similar comments can be made for the other observers we have considered in this paper; see [5] for details.

Fig. 4 compares the performance of the continuous-time and sampled-data implementations of the reduced-order observer in Section IV. The waveform in (a) is just Fig. 1(f) repeated, showing the error in the rotor flux estimate produced by the continuous-time observer, while (b) shows the estimate produced by a sampled-data observer sampling at 10 kHz. It is evident that the sampled-data implementation performs well. For comparison, the waveform in (c) shows the error obtained if one attempts to get away without computing matrix exponentials at all, but simple uses forward differences to approximate derivatives (i.e., uses the "forward Euler" method); the result in this instance is a disaster! Further issues related to microprocessor implementation are discussed in [9], [27], and [28].

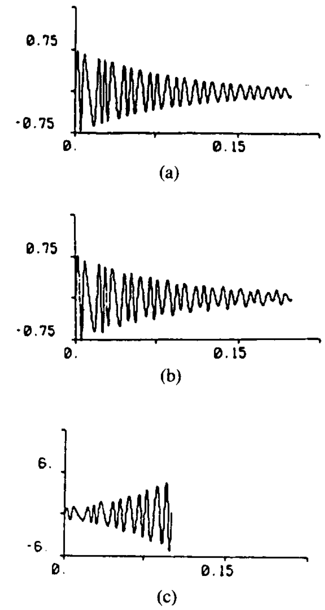

Fig. 4. (a) Same as Fig. 1(f). (b) Error in sampled-data implementation of observer, 10-kHz sampling. (c) Error in approximate sampled-data implementation using forward Euler method.

VIII. Conclusion

This paper has used the perspectives of observer theory to examine the problem of flux estimation in induction machines. Related work in the literature has also been noted. There are several interesting directions in which further studies of state estimation for electrical machines may be usefully pursued. Among these are the development of adaptive observers, extending the work in [1] and [16], and the development of nonlinear observers for estimation of rotor speed as well, extending results such as [19]–[21].

Note added in proof—The interested reader will do well to consult the following very recent work, which extends the results in this paper: 1) Y. Hori, V. Cotter, and Y. Kaya, "A novel induction machine flux observer and its application to a high performance ac drive system," presented at IFAC, 10th World Congress on Automatic Control, Munich, July 1987.

2) V. Cotter, "Implementation of a sampled-data secondary flux observer for an induction motor and minimization of its sensitivity to parameter variation," Master's thesis, Univ. Tokyo, 1987.

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