Optimum Bias Calculation for Parallel Hybrid Switching-Linear Regulators

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***Abstract*- This paper presents an analysis of the bias constraints for switching and linear voltage regulators operated in a parallel-hybrid configuration. Particular emphasis is given to polar and envelope-tracking RF power amplifier (PA) applications requiring wideband dynamic voltage regulation. Ideal expressions are derived for the optimum current contribution of the switching regulator under quasi-static operating conditions. In contrast to previous work, it is shown that the optimum mean current contribution of the switching regulator is not necessarily the DC current to the load, but is a function of the DC and dynamic characteristics of the regulated output voltage. Theoretical maximum efficiency is derived for envelope waveforms that result from two-tone and sinusoidal amplitude modulation of the RF carrier.**

I. Introduction

Hybrid combinations of linear and switching regulators have been proposed both for audio amplifiers [1, 2], and dynamic supply modulators for RF power amplifiers (RF PAs) [3-5]. In contrast to conventional voltage regulation applications, audio and RF-PA supply applications require fast, accurate, time-varying regulation of the output voltage to meet strict spectral performance requirements [6, 7]. Hybrid topologies show promise for these applications because they combine favorable aspects of both switching and linear regulators. Specifically, high gain-bandwidth linear regulators can provide fast voltage regulation and high dynamic range [8], while properly designed switching regulators can achieve high efficiency for a wide range of conversion ratios [9]. As will be described in this work, in many situations the control and bias strategy must be carefully considered in order to achieve significant efficiency advantages over pure linear regulators.

Fig. 1 shows a schematic representation of a hybrid regulator consisting of parallel linear and switching stages. The linear regulator is typically used as a voltage follower with local feedback to reduce output impedance and improve accuracy. With high closed loop bandwidth, the linear regulator can be used to attenuate switching harmonics from the DC-DC converter stage [1, 2], and can also supply a portion of the dynamic power of the signal [5]. In work reported in the literature, the switching regulator forces the average or DC linear regulator current to zero with linear (proportional-integral) control, [3], or hysteretic control, [4]. The switching regulator can also have a non-zero bandwidth and supply some of the dynamic power. A method to determine the optimum bandwidth of the switching stage through simulation is proposed in [5].

In this work, we study the parallel linear-switcher combination for applications related to dynamic power supplies for RF power amplifiers. We expand on the work in [3-5] with particular emphasis on the optimum bias condition. In our approach we assume that the switching regulator operates as a quasi-static current source. This assumption is valid if the envelope frequency is much higher than the bandwidth of the switching regulator. Based on the conduction angle of the linear stage we derive expressions for the optimum switching regulator current as a function of the supply voltage, the average output voltage, and dynamic characteristics of the envelope signal. It is shown that there is an optimum efficiency for such a configuration, and that the optimum switching regulator current varies with the power of the signal. We verify some of the conclusions of [4] and [5], but show that a more efficient control methodology is possible. Importantly, in contrast to previous work, we show that for maximum efficiency, the optimum switching regulator current may be greater than the DC current to the load. We verify our predictions with measured data and propose an adaptive control algorithm to implement our efficiency optimization procedure.

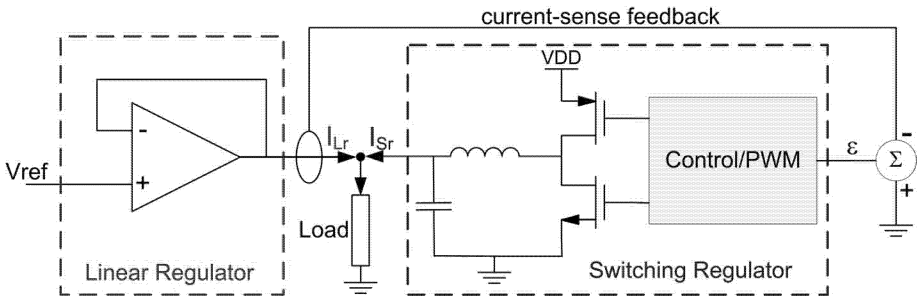

Fig. 1 Traditional parallel-hybrid configuration of linear and switching voltage regulators

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II. Optimum Bias Point: Model and Calculation

To reduce voltage ripple, switching regulators must tradeoff transient response and/or efficiency by increasing the size of the filter components or increasing the switching frequency [10, 11]. Linear regulators, on the other hand, may provide fast transient response and high dynamic range, but are inefficient, especially at low conversion ratios. The hybrid topology can decouple efficiency from transient response and voltage ripple, allowing the performance of a linear regulator with less power consumption.

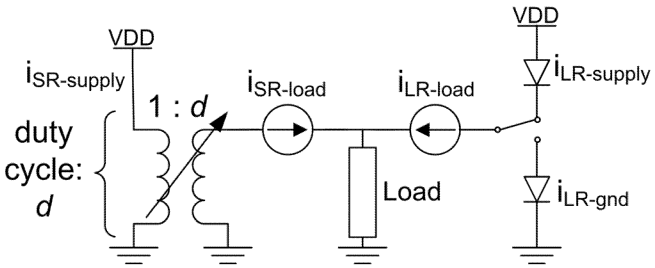

Fig. 2. Proposed hybrid switching regulator model

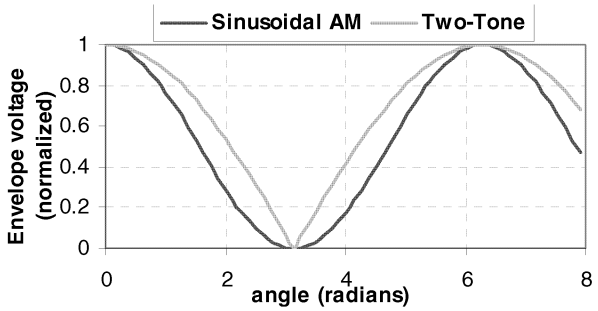

Fig. 3. Envelope waveforms: Sinusoidal AM and 2-tone modulation

In this treatment, we model a step down (buck) DC-DC converter as a quasi-static current source with average conversion ratio, *d*, between the load current and the current drawn from the supply. As shown in Fig. 2, it is assumed that the switching regulator operates as an ideal transformer and has zero bandwidth over some finite time window. The average voltage across the inductor must be zero, so duty cycle is constrained to be the ratio of the average output voltage to supply voltage:

$$d = \frac{\langle v_{out} \rangle}{V_{dd}}, \tag{1}$$

where $\langle V_{out} \rangle$ is the average output voltage over some time window $a \le t \le b$, and $\langle f(t) \rangle = \frac{1}{b-a}\int_a^b f(t)dt$. For periodic *f(t)*, *a* and *b* may be taken as *nT* and *(n+1)T* for integer *n*.

The linear regulator is modeled as an ideal class B topology, with a push-pull rail-to-rail output stage. It can be verified that this is typically the most efficient output stage for signals of interest [12]. With a push-pull output stage, all current sourced to the load comes from the supply; all current drawn from the load sinks to ground. The conduction angle of the linear regulator will be defined as the radial angle in degrees that the linear regulator draws current from the supply for a periodic waveform. The switching regulator can source any proportion of the average current. This allows the linear regulator to operate with any conduction angle between 0-360 degrees. The optimum bias point and conduction angle are derived based on average efficiency,

$$\langle \eta \rangle = \frac{\langle P_L \rangle}{\langle P_S \rangle}, \tag{2}$$

where $\langle P_L \rangle$ is the average power to the load and $\langle P_S \rangle$ is the average power from the supply. Assuming an ideal situation as in Fig. 2, the average power from the supply follows as:

$$\langle P_S \rangle = V_{DD} \cdot \left[\langle i_{LR} \rangle + \langle i_{SR} \rangle \cdot d \right], \tag{3}$$

where i_{LR} and i_{SR} are the linear and switching regulator currents delivered to the load, *d* is the duty cycle, and V_{DD} is the supply or battery voltage. It should be noted that (3) assumes an ideal switching regulator with no loss. However, the average efficiency concept extends to real switching and linear regulator components that have various non-idealities such as non-zero switching and conductive losses. Also, this work assumes that the load is linear and resistive, whereas realistic power amplifier loads may be nonlinear and reactive. This can complicate the calculation of average efficiency, but does not reduce the utility of the optimization procedure. To calculate average efficiency it is necessary to derive or measure average currents, $\langle i_{LR} \rangle$ and $\langle i_{SR} \rangle$. For simple envelope waveforms, such as sinusoidal AM and two-tone RF signals, expressions for (2) and (3) can be derived explicitly based on characteristics of the regulated voltage signal and the supply or battery voltage. For cellular and wireless internet standards, such as CDMA, UMTS and the 802.11 standards, hand calculations are difficult due to the non-periodic nature of the envelope waveform. However, assuming $\langle i_{LR} \rangle$ and $\langle i_{SR} \rangle$ can be measured, the optimum bias point can still be determined, as will be presented in section IV.

III. Efficiency Optimization: Sinusoidal and Two-Tone Carrier Modulation

Envelope signals that result from explicit modulation of the RF carrier may allow direct solution of optimum biasing expressions. For the case of sinusoidal amplitude modulation (AM) of the RF carrier, the envelope voltage and current waveforms may be written as

$$vo = vdc + va \cdot \cos(wt), \text{ and}$$
$$io = idc + ia \cdot \cos(wt). \quad (4)$$

Here, *vo* and *io* are the output voltage and current respectively, *va* and *ia* are the voltage and current amplitudes, and *vdc* and *idc* are the DC values. In (4), and in the rest of this work, the load is assumed to be resistive. To derive an expression for the average power drawn from the supply, as in (3), the value for $\langle i_{LR} \rangle$ is solved by assuming that i_{SR} is constant during the period of the envelope signal, and finding the corresponding conduction angle, Φ, that the linear regulator conducts current from the supply. The linear regulator current is reduced by the switching regulator current, so for the waveform in (4), Φ can be written as

$$\Phi = \cos^{-1}\left(\frac{i_{SR} - idc}{ia}\right). \quad (5)$$

Based on (5), the average linear regulator supply current can be written in terms of conduction angle as

$$\langle i_{LR} \rangle = \frac{1}{2\pi}\int_{-\Phi}^{\Phi}(idc + ia \cdot \cos(\phi) - isr)d\phi$$
$$= \frac{ia}{\pi}[\sin\Phi - \Phi\cos\Phi]. \quad (6)$$

Here, the average linear regulator current is only a function of the amplitude of the load current swing and the conduction angle. The average efficiency for the ideal system, as in Fig. 2, can be written in terms of the properties of the envelope waveform using (1), (3), and (6), specifically noting that the current the switching regulator draws from the supply is reduced by conversion ratio, *d*. Average efficiency for the sinusoidal-AM envelope follows as

$$\langle \eta \rangle = \frac{vdc \cdot idc + \frac{va \cdot ia}{2}}{i_{SR} \cdot vdc + V_{dd}\frac{ia}{\pi}[\sin\Phi - \Phi\cos\Phi]}. \quad (7)$$

In (7), average efficiency is written purely in terms of properties of the envelope waveform, the supply voltage, and the conduction angle, Φ, of the linear regulator. Substituting (5) into (7), average efficiency is expressed as a function of the current supplied by the switching regulator, i_{SR}. Using this result, an algebraic minimization can be done to find the switching regulator current that provides maximum average efficiency. This is the solution to $\frac{d\langle\eta\rangle}{di_{SR}} = 0$, and can be found as

$$i_{SR}^{*} = idc + ia \cdot \cos\left[\pi\frac{vdc}{V_{dd}}\right], \quad (8)$$

where i_{SR}^{*} is the optimum switching regulator current. The optimum average efficiency, $\langle\eta\rangle^{*}$, is expressed as

$$\langle\eta\rangle^{*} = \frac{vdc \cdot idc + \frac{va \cdot ia}{2}}{vdc \cdot idc + V_{dd}\frac{ia}{\pi}\sin(\Phi^{*})}, \quad (9)$$

where Φ^{*} is the optimum conduction angle for the linear regulator, and can be written as

$$\Phi^{*} = \pi\frac{vdc}{V_{dd}}. \quad (10)$$

As seen in (8), the optimum current supplied by the switching regulator is not necessarily equal to the DC current supplied to the load, but is in fact a function of the DC and dynamic characteristics of the envelope signal as well as the supply voltage. This is a departure from the control schemes presented in [3-5], where the mean switching regulator current is the DC load current.

The calculation is similar for two-tone signals except that different expressions are obtained for the conduction angle and optimum biasing conditions. For the case that the RF carrier consists of two tones with different frequencies, w_1 and w_2, but equal magnitudes, *va*, the envelope is a full-wave rectified sinusoid with a peak value of *2*va*, as in [5]:

$$v_{env} = \left|2va\cos\left(\frac{w_2 - w_1}{2}t\right)\right|. \quad (11)$$

In this case, the conduction angle as a function of the switching regulator current is

$$\Phi = \cos^{-1}\left(\frac{i_{SR}}{2ia}\right), \quad (12)$$

where $ia = \frac{va}{R_{load}}$ is the amplitude of the current swing of one of the two tone signals.

Following a similar procedure to the sinusoidal envelope signal, the optimum quasi-static switching regulator current contribution for the two-tone case is

$$i_{SR}^{*} = 2 \cdot ia \cdot \cos(\Phi^{*}), \quad (13)$$

where V_{dd} is the supply or battery voltage. In this case, the optimum conduction angle is $\Phi^{*} = \frac{2va}{V_{dd}}$. The maximum average efficiency simplifies to:

$$\langle\eta\rangle^{*} = \frac{\pi}{2}\frac{va}{V_{dd} \cdot \sin(\Phi^{*})}. \quad (14)$$

For the two-tone case, ideal output efficiency is bounded between 93.3% for rail-rail modulation, and 78.5% (pi/4) as $va \to 0$. In theory if the switching regulator supplied the DC current, efficiency would be bounded by 92.7% to 0%.

This range fits with the efficiency of 87% for a two-tone envelope as reported in [5].

IV. Experimental Results and Comparison to Theory

A prototype was created to verify the biasing model and predictions for maximum efficiency. The prototype was done at the board level with discrete components. A fast operational amplifier driving a class-B common-collector buffer stage was used for the linear regulator. The switching regulator was controlled as a current source using a large, low-loss inductor. A *10Ω* resistive load was used to model the PA.

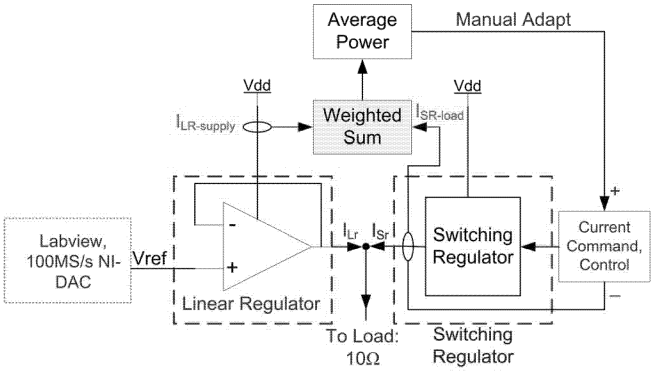

Fig. 4 Experimental Setup

To calculate average efficiency, the supply current feeding the output stage of the linear regulator was measured as in Fig 4. The experimental efficiency, to be compared to (9) and (14), was calculated as the sum of the average linear regulator current from the supply and the weighted output current of the switching regulator. The switching regulator current was weighted by a factor, *d*, as in (1) to reflect lossless DC-DC voltage conversion. Lossless conversion was assumed to verify the ideal maximum efficiency case. To further reflect the ideal case, the bias current of 6mA of the linear regulator was not included in the calculation.

As shown in Fig. 4, the input signal representing the dynamic envelope trajectory was delivered by a National Instruments D/A converter running at 100MS/s. The Labview software interface was used to generate and supply envelope waveform signals with up to a 20MHz bandwidth. The switching regulator current command was adjusted manually to achieve the highest average efficiency for a given input signal.

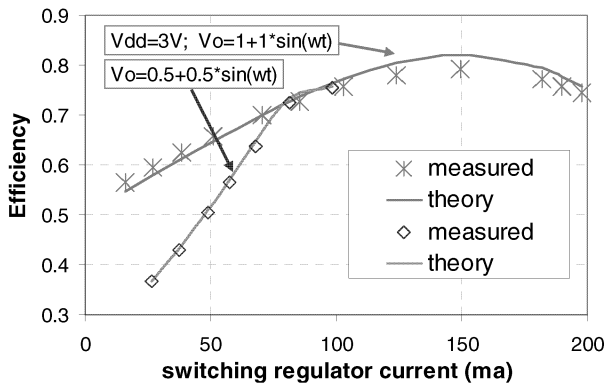

Fig. 5. Average efficiency vs switching regulator current contribution

Fig. 5 shows how average efficiency varies with the switching regulator current contribution for 2 amplitudes of the sinusoidal-AM case. In both cases the measured data track the predictions of (8) which are plotted for comparison in the figure. Some discrepancy between theory and measurement is caused by additional sources of loss that are not modeled such as base current in the bipolar output stage of the linear regulator. Base current may be a source of error since it can add or subtract from current delivered to the load, but is not reflected in the measurement of the output stage supply current, which is taken at the collector of the high-side transistor. Overall, measured values of i_{SR}* were in good agreement with theory, but to fine-tune the measured maximum efficiency it was necessary to manually adapt i_{SR} to achieve optimum performance.

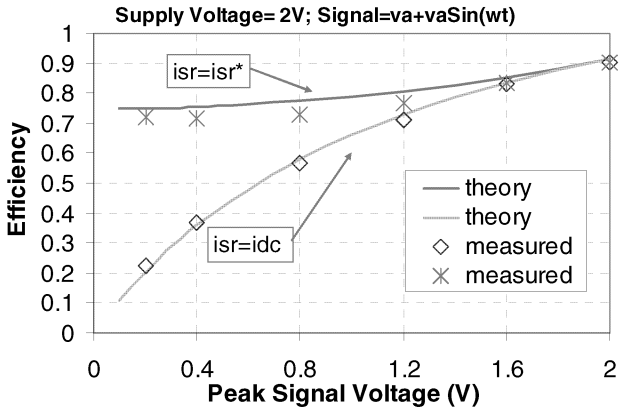

Fig. 6 Sinusoidal-AM modulation: Average efficiency vs envelope modulation amplitude, optimum and traditional methods

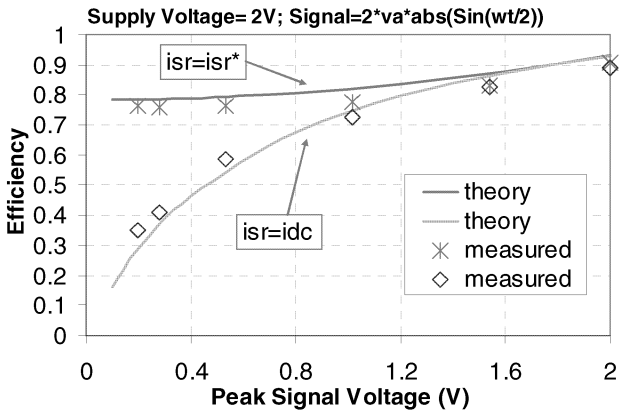

Fig. 7 Two-tone modulation: Average efficiency vs envelope modulation amplitude, optimum and traditional methods

Fig. 6 shows average efficiency for a sinusoidal-AM modulated carrier as the signal amplitude is reduced. The supply voltage was set to 2V, and the signal amplitude, *va*, from (4), was swept from rail-rail swing to nearly zero amplitude. In all cases, the sinusoidal envelope signal was set such that *va*=*vdc*, corresponding to full modulation of the RF carrier.

In the traditional scheme, if the switching regulator contributes the DC load current, average efficiency falls to 0% as the modulation amplitude is reduced. However, if the switching regulator supplies the optimum current, i_{SR}*, as derived in (8), the average efficiency can be kept between 75% and 91.7% across the entire range of operation. Intuitively, this can be explained based on the operation of the linear regulator:

- At high output swing levels, the switching regulator supplies the DC current. In this case the linear regulator operation approaches class B, or 180° conduction angle for each push-pull output device. This causes the curves in Fig. 3 to converge for high amplitudes.

- At low output swing the switching regulator sources more than the DC current, such that, for the sinusoidal-AM case, $\lim_{va\to 0} i_{SR} = idc + ia$. This means that the high side pass transistor operates in class-C with the transistor conducting for less than 180°. Alternatively, the low side pass transistor approaches class-A operation, drawing current only from the switching regulator output.

Similar results are shown in Fig. 7 for 2-tone modulation waveforms. Here, the supply voltage was 2 V and the amplitude parameter, *va*, from (11), was swept between 1 V and 50 mV. Like the sinusoidal-AM waveforms in Fig. 6, if the switching regulator current was set such that i_{SR}=i_{DC}, efficiency falls towards 0% as amplitude is reduced. However, if the switching regulator current follows the optimum trajectory, i_{SR}=i_{SR}*, then theoretical maximum efficiency stays above 78.5% across the range of operation.

Overall, the measured data in Fig. 6 and Fig. 7 are in good agreement with the predictions of (9) and (14). Additional sources of loss, including losses due to base current and interconnect cause some discrepancy between measurement and theory. Uncertainty in power measurements leads to further deviation at lower amplitudes. An additional source of error is tolerance in setting the switching regulator bias current, to which average efficiency becomes increasingly sensitive at low amplitudes.

V. CDMA/802.11 Envelope Waveforms

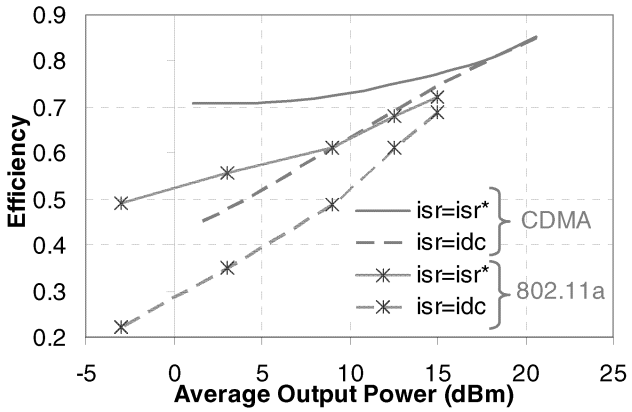

Fig. 8 Measured average efficiency vs output power: IS-95 CDMA, 802.11a WLAN

Fig. 8 shows measured results for CDMA and IEEE-802.11a supply modulation waveforms. These waveforms were generated with the Agilent *Advanced Design Systems* (ADS) software and converted to a format suitable for data conversion. The waveforms were delivered to the hybrid regulator with the National Instruments DAC as shown in Fig. 4. The x-axis shows *average power* delivered to the 10Ω load. In this set of experiments the maximum *average power* delivered in the CDMA waveforms was just over 20dBm. For the 802.11a waveforms average power was 15dBm. The higher peak-average power ratio (PAPR) of the 802.11a standard is the principal reason that average efficiency is lower than the other waveforms. Higher PAPR indicates that the dynamic output voltage tends to deviate more from the average voltage. This causes the linear regulator to draw more current from the supply, reducing average efficiency compared to low PAPR signals.

Importantly, Fig. 8 shows that significant power savings are possible if the switching regulator supplies the optimum current, i_{SR}^*, rather than the DC current to the load. The power savings are most dramatic at low output power levels, when the average output voltage is significantly less than the supply voltage. This may result in dramatic improvements in the average efficiency of cellular and

WLAN systems, which tend to operate at significantly less than maximum output power [13]. At low power, the switching regulator supplies more than the DC current. This reduces the net current that the linear regulator draws from the supply. A practical solution for future designs may be to adaptively seek i_{SR}^{*} with an extremum-seeking adaptive control architecture, such as is presented in [9] for dead-time optimization. An adaptive architecture could achieve maximum efficiency for a wide range of output power, maintain robustness against temperature and process variation, and exploit the convex profile of the efficiency versus switching regulator current contribution.

VI. Conclusion

We have presented a biasing scheme for hybrid voltage regulators consisting of linear and switching stages operated in parallel. It was shown that there is an optimum current contribution from the switching stage that is not equal to the DC current to the load. Instead, optimum efficiency is a function of the DC and dynamic characteristics of the regulated voltage signal as well as the supply voltage. Our analysis implies a highly practical biasing scheme where the switching regulator operates with only a modest bandwidth, enabling a low cost solution with high efficiency, and potentially eliminating extra series dc-dc conversion stages between the battery and the PA supply voltage. Overall this work demonstrates a valuable practical and theoretical limit for the design of dynamic voltage regulators.

Acknowledgment

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VII. References

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