

# Lyapunov-Based Control for Switched Power Converters

Seth R. Sanders and George C. Verghese

**Abstract**—Beginning with fundamental properties such as passivity or incremental passivity of the network elements comprising a switched power converter, the nominal open-loop operation of a broad class of such converters is shown to be stable in the large via a Lyapunov argument. The obtained Lyapunov function is then shown to be useful for designing globally stabilizing controls that include adaptive schemes for handling uncertain nominal parameters. Numerical simulations illustrate the application of this control approach in dc-dc converters.

## I. INTRODUCTION

**M**OST control schemes for power electronic circuits in present use are obtained by linearizing a nonlinear model about a nominal operating point or trajectory. Large-signal transients that occur at power up or overload recovery are handled in an ad hoc manner. Designers analyze each circuit individually to prescribe a scheme to accommodate a designated set of large-signal transients. This paper addresses the issue of how one might do better. In particular, the paper develops a methodology for designing control laws for fast-switching converters that result in globally stable behavior, robustness against parametric uncertainty, and satisfactory transient response. The main approach in this paper is based on the use of Lyapunov functions.

There has been considerable previous work along these lines. The paper of Erickson *et al.* [11] develops a large-signal averaged model for switched converters and points out the hazards of control designs based on small-signal linearized models. The previous work on large-signal control schemes can be divided roughly into two groups. One is based on so-called "switching-law" controls where the position of a controlled switch is directly commanded as a function of the instantaneous values of the circuit variables. Examples of these are the sliding mode control schemes of [1]–[7], [21] (which include current-mode control) and the bang-bang control schemes of [9], [10],

[15]. The second approach relies on the state-space averaged model for the converter of interest. Representative schemes are those in [12]–[14], [22]. References [12] and [14] also consider control design using Lyapunov functions.

The paper is organized as follows. Section II develops fundamental stability properties for a broad class of switching converters. In particular, we derive the form of a Lyapunov function that illustrates that each member of this class is open-loop stable. The Lyapunov-based control approach is introduced in Section III. That section includes a simple example to demonstrate the method, an outline of the general approach, and an illustration of how an adaptation scheme can be incorporated to handle uncertainties in the nominal operating point.

## II. OPEN-LOOP STABILITY OF SWITCHING CONVERTERS

In this section, switching converter systems (understood to include source and load) that consist of an interconnection of ideal dc sources, ideal switches, incrementally passive resistors, and passive linear reactive elements are considered. Multiport circuit elements are included in the development here. This class of switching converters is shown to be stable by exhibiting a Lyapunov function that corresponds to the *energy in the increment* with respect to an arbitrary nominal state trajectory. The argument is extended to include nonlinear reactive elements that are strictly relatively passive in the case where the switching frequency becomes infinite and stability with respect to an equilibrium point is considered. Essential background on network theoretic issues for the development in this section is given in Appendix A. (Also, see [16]–[18].)

### A. Switching Converter Stability Under Finite Switching Frequency

Let the switching converter be composed of ideal dc sources, ideal switches, incrementally passive resistors, and linear passive reactive elements. A diode may be considered as either an ideal switch or as an incrementally passive resistor. We suppose the switches are operated in accord with a given arbitrary switching pattern and suppose that we are given a nominal solution corresponding to the given switching pattern.

For each branch of the network, denote the nominal trajectory by  $\{\bar{v}(t), \bar{i}(t)\}$ , and form the (not necessarily

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S. R. Sanders is with the Department of Electrical Engineering and Computer Sciences, University of California, Berkeley, CA 94720.

G. C. Verghese is with the Laboratory for Electromagnetic and Electronic Systems, Massachusetts Institute of Technology, Cambridge, MA 02139.

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small) *increments* with respect to the nominal trajectory for each network branch, i.e.,

$$\begin{aligned}\delta i(t) &= i(t) - \tilde{i}(t) \\ \delta v(t) &= v(t) - \tilde{v}(t)\end{aligned}\quad (1)$$

By applying Tellegen's theorem to the increments in all the network branches when the circuit is in any one of its topologies, we obtain

$$\begin{aligned}0 &= \sum_{\text{dc sources}} \delta i \delta v + \sum_{\text{Switches}} \delta i \delta v + \sum_{\text{Res.}} \delta i \delta v \\ &+ \sum_{\text{Ind.}} \delta i \delta v + \sum_{\text{Cap.}} \delta i \delta v.\end{aligned}\quad (2)$$

The summation involving dc sources is always zero as the increment in either voltage or current of each term is necessarily zero. The terms involving switches also add zero contribution to the sum in (2) for the same reason. The third summation on the right-hand side of (2) is always nonnegative as each term is individually positive or zero as a result of the incremental passivity of the resistors. In fact, this summation can be thought of as the dissipated *power in the increment* with respect to the given nominal trajectory. The remaining two summations represent the time rate of change of the *stored energy in the increment* as

$$\frac{d}{dt} V(\delta x) = \sum_{\text{Ind.}} (\delta i_k)^* (\delta v_k) + \sum_{\text{Cap.}} (\delta v_k)^* (\delta v_k) \quad (3)$$

where

$$\begin{aligned}V(\delta x) &= \sum_{\text{Ind.}} \left(\frac{1}{2}\right) (\delta i_k)^* L_k (\delta i_k) \\ &+ \sum_{\text{Cap.}} \left(\frac{1}{2}\right) (\delta v_k)^* C_k (\delta v_k)\end{aligned}\quad (4)$$

and  $L_k$  and  $C_k$  represent inductance and capacitance matrices, respectively, for multiport coupled inductances and capacitances. (Note that a particular  $L_k$  or  $C_k$  may be a scalar if the corresponding element is a two-terminal reactive element. Also note that the superscript \* indicates transpose.) Because of the assumption on passivity of the reactive elements, the quantity  $V(\delta x)$ , which we refer to from now on by the suggestive name *energy in the increment*, is a positive definite quadratic function of the incremental state variables. Because (2) holds identically for any of the possible circuit topologies, the energy in the increment is a Lyapunov function for the given nominal trajectory. In particular, we have

$$\frac{d}{dt} V(\delta x) = - \sum_{\text{Res.}} \delta v \delta i \leq 0. \quad (5)$$

In conclusion, the energy in the increment is a Lyapunov function for the given nominal trajectory, and we see that the nominal trajectory is stable in the large. Because the nominal trajectory selected above can be taken as any solution trajectory, this statement implies that any two solution trajectories do not diverge.

Typically, asymptotic stability in the large can be concluded as well as at least some parasitic loss is always

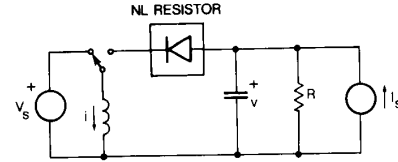


Fig. 1. Up-down converter redrawn to illustrate stability in case of discontinuous conduction

associated with each energy storage element, i.e., series resistance with inductors and parallel leakage resistance with capacitors. An argument for asymptotic stability appears in [16] for circuits that have a dc equilibrium point and consist of only two-terminal elements. In [16], lossiness is guaranteed to be associated with each state variable by excluding inductor-capacitor-voltage source loops and inductor-capacitor-current source cutsets. (In this case, we would require that all resistors be strictly incrementally passive.)

A special case of the above result is when the switches are operated with a periodic switching pattern and there exists a nominal periodic steady-state solution. In this case, the result states that the given periodic steady-state trajectory is stable in the large. This result is of particular interest for the case of a dc-dc converter operating with constant switching frequency. Note that this result holds for dc-dc converters operating in the *discontinuous* conduction mode. This can be seen by redrawing the schematic for the dc-dc converter of interest with an ideal single-pole double-throw (SPDT) switch and an incrementally passive resistive device (i.e., a diode) replacing each transistor-diode pair. For example, we would redraw the up-down converter of Fig. 2 as shown in Fig. 1. The circuit of Fig. 1 satisfies the conditions for its nominal periodic trajectory to be stable in the large, and it makes no difference whether or not the nominal trajectory contains a portion where the inductor current is identically zero.

### B. Stability under Infinite Switching Frequency and Constant Duty Ratio

The aforementioned theory can be extended to the case where the switching frequency becomes infinite. This case is naturally analyzed with the state-space averaged model for a converter, and in this way an open-loop stability result can be obtained for state-space averaged models. The main difference from the case of finite switching frequency is that one needs to consider the stability of an equilibrium point for an averaged model of a dc-dc converter rather than a limit cycle. In such a setup, it is possible to include nonlinear reactive elements as well as nonlinear resistive elements. This is of interest in power electronic circuits because nonlinear reactive elements do occur in practice. The following theorem summarizes the result for infinite switching frequency.

**Theorem 2.1:** Suppose that a switching converter is constructed from ideal switches, ideal dc sources, incrementally passive resistors, and reactive elements that are

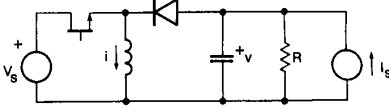


Fig. 2. Up-down converter.

strictly relatively passive and that its averaged model has an equilibrium point, then the equilibrium is stable in the large.

This theorem is easily proven by demonstrating the existence of an appropriate averaged circuit model. See [23] and [24] for details. We rely heavily on this result in the sequel where control laws based on state-space averaged models are developed.

### III. LYAPUNOV-BASED CONTROL DESIGN

In this section, an approach to control of switching power converters based on the use of Lyapunov functions will be introduced. The main focus will be on control design based on the state-space averaged model for a given switching converter. The converters of interest are those that satisfy the conditions guaranteeing that nominal state trajectories are globally stable under open-loop operation, specifically converters constructed from incrementally passive resistors, ideal sources, ideal switches, and passive linear reactive elements. One particular choice of Lyapunov function for control design purposes that will be of interest is the energy in the increment.

We begin by illustrating the Lyapunov-based control method with an application to an up-down converter. Then, we demonstrate how such a control design can be obtained in a more general way. There is typically some freedom in the choice of Lyapunov function for the control design, but we exhibit some particular advantages of using the energy in the increment. Finally, we show how an adaptation scheme can be incorporated to handle parametric uncertainty. Generalizations to converters containing nonlinear circuit elements, to converters that handle time-varying input-output waveforms, and to converters operating in the discontinuous conduction mode are given in the thesis [23]. A method (dual to the control design approach) for designing state observers is also considered in [26].

#### A. Example: Up-Down Converter

Consider the up-down converter of Fig. 2, which has a state-space averaged model of the form

$$x' = Ax + (Bx + b)d \quad (6)$$

where the two-component state  $x$  consists of the deviation of the inductor current from its nominal value ( $x_1 = i - i_n$ ) and the deviation of the capacitor voltage from its nominal ( $x_2 = v - v_n$ ), and where the input  $d$  is the deviation in the duty ratio from its nominal value ( $d = d_i - d_n$ ). (Note that  $d_i$  indicates the total duty ratio here.) The parameter values listed below were selected for operation at a switching frequency of 50 kHz.

$$\begin{aligned} C &= 5.4 \mu\text{F} \\ L &= 0.18 \text{ mH} \\ R &= \infty \\ d_n &= 3/8 \\ V_s &= 15 \text{ V} \\ I_s &= 2 \text{ A} \\ v_n &= -9 \text{ V} \\ i_n &= 3.2 \text{ A} \end{aligned}$$

The relevant matrices of the system are as follows:

$$\begin{aligned} A &= \begin{bmatrix} 0 & (1 - d_n)/L \\ -(1 - d_n)/C & 0 \end{bmatrix} \\ B &= \begin{bmatrix} 0 & -1/L \\ 1/C & 0 \end{bmatrix} \\ b &= \begin{bmatrix} (V_s - v_n)/L \\ i_n/C \end{bmatrix} \\ Q &= \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix} \end{aligned}$$

The result on open-loop stability in Section II guarantees that the energy in the increment is a Lyapunov function for open-loop operation of this circuit. For the up-down converter, the energy in the increment takes the form

$$V = \frac{1}{2}L(i - i_n)^2 + \frac{1}{2}C(v - v_n)^2 \quad (7)$$

or

$$V = \frac{1}{2}x^*Qx. \quad (8)$$

Differentiating  $V$  along the system trajectories, we obtain

$$\begin{aligned} \frac{d}{dt} V(x) &= \frac{1}{2}x^*(QA + A^*Q)x \\ &+ \frac{1}{2}\{x^*(QB + B^*Q)x + 2b^*Qx\}d. \quad (9) \end{aligned}$$

It turns out that  $QA + A^*Q = 0$  for this example, which verifies that the energy in the increment is a Lyapunov function for open-loop operation ( $d = 0$ ). In this example, it is also true that  $QB + B^*Q = 0$ . These relationships hold because of the lossless nature of the example converter, i.e., the lack of resistive elements in the converter. Considering these relationships, (9) simplifies considerably to

$$\frac{d}{dt} V(x) = (b^*Qx)d. \quad (10)$$

Many stabilizing control schemes can be obtained by inspection of (10). We consider the simple control law  $d = -\alpha b^*Qx$  with  $\alpha$  real and positive, modified to handle the

duty ratio saturation constraint  $-d_n \leq d \leq 1 - d_n$ , i.e.,

$$d = \begin{cases} -\alpha y, & -d_n \leq -\alpha y \leq 1 - d_n \\ -d_n, & -\alpha y < -d_n \\ 1 - d_n, & -\alpha y > 1 - d_n \end{cases} \quad (11)$$

where  $y = b^*Qx$ . Here, the variable  $y$  takes the form

$$\begin{aligned} y &= (V_s - v_n)(i - i_n) + i_n(v - v_n) \\ &= (V_s - v)(i - i_n) + i(v - v_n). \end{aligned} \quad (12)$$

Note that the only dependence on circuit parameters is on the nominal value of the inductor current, the capacitor voltage, and the source voltage. This property is shared by analogous control schemes based on the energy in the increment for many other switching converters, as will be discussed in Section III.C. The dependence on nominal values of circuit variables is of crucial importance, and this issue is addressed in Section III.D. There, a method for adaptively estimating these values is developed.

To investigate the closed-loop behavior, we examine the derivative of the Lyapunov function  $V(x)$  along the closed-loop system trajectories:

$$\begin{aligned} \frac{d}{dt} V(x) &= yd \\ &= \begin{cases} -\alpha y^2, & -d_n \leq -\alpha y \leq 1 - d_n \\ -d_n y, & -\alpha y < -d_n \\ (1 - d_n)y, & -\alpha y > 1 - d_n \end{cases} \end{aligned} \quad (13)$$

In the saturated regions (the second and third lines of (13)), the time derivative of  $V(x)$  is strictly negative since either  $V' < -d_n^2/\alpha$  or  $V' < -(1 - d_n)^2/\alpha$ . As a result, state trajectories quickly enter the unsaturated region. In the unsaturated region (the first line in (13)),  $V(x)$  is strictly decreasing if  $y \neq 0$ , and asymptotic stability can be concluded by LaSalle's theorem because  $y \equiv 0$  is not a system trajectory unless  $x \equiv 0$ . To see this, note that  $y \equiv 0$  implies  $d \equiv 0$  and the following:

$$\begin{aligned} b^*Qx &= 0 \\ b^*QAx &= 0 \end{aligned} \quad (14)$$

with the last line in (14) obtained by noting that  $y' = 0$ . The existence of a nonzero solution  $x$  to (14) is equivalent to the statement that the pair  $\{b^*Q, A\}$  is unobservable. However, this pair is observable in this example, and therefore there are no system trajectories that do not converge to the origin.

In this example, we have not considered the effect of lossiness due to parasitic and/or load resistances. The effect of such passive resistances would only enhance our stability result by causing additional nonpositive terms of the form  $-x^*Rx$  (with  $R$  positive semidefinite) to be added to the terms on the right-hand sides of (13).

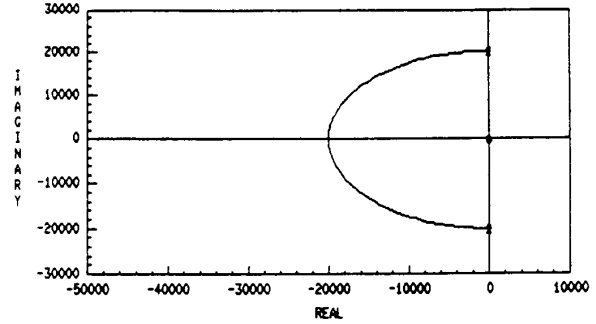


Fig. 3. Root locus for linearized closed-loop control system.

Asymptotically, the decay of the Lyapunov function  $V(x)$  is controlled by the eigenvalues of the small signal model obtained by linearizing the closed-loop system about  $x = 0$ . In this example, there is some freedom in placing the eigenvalues of the linearized closed-loop system by choice of the gain  $\alpha$ . A root locus of the closed-loop eigenvalues of the small-signal model is shown in Fig. 3. To minimize the maximum of the real parts of the eigenvalues, for example, the gain should be selected so that the two eigenvalues coincide on the real axis at  $-20.05$  krad/s. An easy calculation indicates that the value of the gain required to obtain this eigenvalue placement is approximately  $\alpha = 0.00785$ . In the remainder of the discussion of this example, a value of the gain of  $\alpha = 0.008$  is used. The resulting closed-loop eigenvalues are at about  $-16.7$  and  $-24$  krad/s. Note that in this example the dynamical behavior of the small signal closed-loop dynamics is limited by the natural resonant frequency  $(1 - d_n)/\sqrt{LC}$  of the open-loop state-space averaged system. Because the bandwidth of the closed-loop dynamics is usually designed to be approximately an order of magnitude below the switching frequency, and since this is also a typical resonant frequency of the open-loop dynamics for a reasonably designed converter, the preceding limit on attainable closed-loop bandwidth is acceptable.

We expect the closed-loop system to be very well behaved, and this is confirmed by the digital computer simulation shown in Fig. 4. In the following subsection, we present a derivation of a class of control schemes to which the above example belongs.

### B. A Basic Approach to a Lyapunov-Based Control Design

In this section, we show how to derive a class of control laws for a switching converter model of the form (6), to which the example (11) belongs. Note that the open-loop stability of the system (6) is crucial for this approach, and hence we restrict attention to switching converters that satisfy the conditions guaranteeing stability under nominal duty ratio operation. A basic first step in this approach, as illustrated earlier, is the specification of a Lyapunov function for open-loop operation. The model (6) is

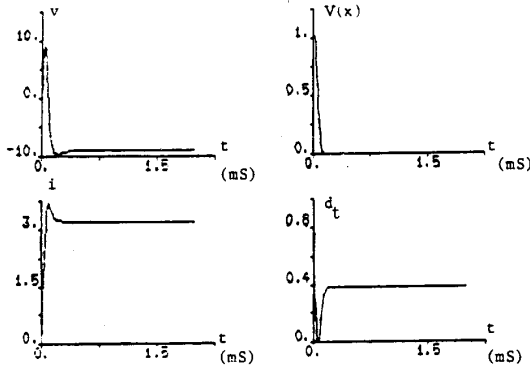


Fig. 4. Digital simulation of up-down converter under nonlinear feedback control scheme.

linear and time-invariant in the case of open-loop operation under a constant nominal duty ratio, i.e.,  $d = 0$ . Because the open-loop model is known a priori to be stable, it is generally possible to determine a family of suitable quadratic Lyapunov functions. In fact, in the case where the matrix  $A$  is asymptotically stable, it is possible to parametrize the family of such quadratic functions with the Lyapunov equation

$$A^*Q_1 + Q_1A = -P_1P_1^* \quad (15)$$

where  $\{P_1^*, A\}$  is an observable pair. The existence of a positive definite, symmetric solution  $Q_1$  to (15) is guaranteed by the stability of the matrix  $A$  and the observability of the pair  $\{P_1^*, A\}$  [8]. See [23] for a method of selecting a suitable matrix  $Q_1$  for the case where the matrix  $A$  has simple eigenvalues on the  $j\omega$  axis.

Having determined the form of a suitable matrix  $Q_1$ , it is straightforward to specify a globally stabilizing control law for the model (6) of the form (11), but based on the Lyapunov function  $V(x) = \frac{1}{2}x^*Q_1x$ , as follows:

$$d = \begin{cases} -\alpha y, & -d_n \leq -\alpha y \leq 1 - d_n \\ -d_n, & -\alpha y < -d_n \\ 1 - d_n, & -\alpha y > 1 - d_n \end{cases} \quad (16)$$

where  $y = (Bx + b)^*Q_1x$ . One particular choice for  $Q_1$  is  $Q$  where  $V(x) = \frac{1}{2}x^*Qx$  is the energy in the increment, and it turns out that this choice leads to certain nice features, which are elaborated below.

### C. Advantages of the Use of the Energy in the Increment for Control Purposes

As noted in Section III-B, there is typically some freedom in the choice of the Lyapunov function that can be used in the control designs described there. Here, we outline three advantages obtained by using the energy in the increment as the Lyapunov function in these control schemes. One advantage of the choice of the energy in the increment as the Lyapunov function for control design purpose arises in the computation of the variable  $y = (Bx$

$+ b)^*Qx$  that is used in these control schemes. In particular, one can always measure the vector  $Q(Bx + b)$ . To see this, consider the modification of (6) where we multiply this equation on the left by the matrix  $Q$ , giving

$$Qx' = QAx + Q(Bx + b)d. \quad (17)$$

Now the vector on the left-hand side of (17) is composed of the time derivatives of the inductor fluxes and the time derivatives of the capacitor charges. The elements of this vector are necessarily inductor voltages and capacitor currents. The vector  $Q(Bx + b)$  is the amount by which this vector changes when the duty ratio steps from  $-d_n$  to  $1 - d_n$ , or equivalently, the amount this vector changes when the switch configuration is changed. In general, it is possible and feasible to determine the vector  $Q(Bx + b)$  during each cycle. To do this, for each inductor branch one would measure the voltage across the branch in each of the two switch configurations and then form the difference of the two measurements. This difference constitutes the element of  $Q(Bx + b)$  corresponding to the particular inductor port. In the case of a capacitor, one would measure the current flowing into the capacitor in each of the two switch configurations and form the difference of the two measured currents. This difference constitutes the element of  $Q(Bx + b)$  corresponding to the particular capacitor. By performing the described measurement process, it is possible to obtain an accurate measurement of the vector  $Q(Bx + b)$ . Consequently, one can compute the variable  $y = x^*Q(Bx + b)$  by forming the inner product of  $x$  and  $Q(Bx + b)$ . The only parametric dependence is therefore on the nominal state values required to determine  $x$ , the deviation in the states from their nominal values.

In certain cases, it is possible to further simplify the measurement of  $Q(Bx + b)$ . For these cases, it is possible to directly measure the vector  $Q(Bx + b)$  by measuring certain branch voltages and branch currents in the circuit at one time instant. One such example is the up-down converter of Fig. 2. For this example, the vector

$$Q(Bx + b) = \begin{bmatrix} V_s - v \\ i \end{bmatrix} \quad (18)$$

and these quantities can be directly measured on the converter circuit. Necessary and sufficient conditions for such a simple measurement of the vector  $Q(Bx + b)$  are given in [23].

A second potential advantage of the choice  $Q_1 = Q$  in (16) is that it is possible to use a nearly linear version of this control algorithm by replacing  $y = (Bx + b)^*Qx$  in (16) with  $y_{lin} = b^*Qx$ , and still maintain global stability. Of course, the saturation constraints are still in effect. To see that global stability is maintained, consider the following Lyapunov analysis with  $V(x) = \frac{1}{2}x^*Qx$ :

$$\begin{aligned} \frac{d}{dt}V(x) &= \frac{1}{2}x^*[(A + dB)^*Q \\ &\quad + Q(A + dB)]x + (b^*Qx)d. \end{aligned} \quad (19)$$

Now the first term on the right-hand side of (19) is always nonpositive. This follows from the fact that the energy in the increment takes the form  $\frac{1}{2}x^*Qx$  for any nominal duty ratio, with the fixed matrix  $Q$ . The choice of the control in (16) (using  $y_{lin}$ ) forces the second term on the right-hand side of (19) to be nonpositive. Global stability results from the nonpositivity of the right-hand side of (19). Hence, the choice of  $Q_1 = Q$  in (16) permits the use of a feedback control that requires only the computation of the linear variable  $y_{lin}$ .

A third advantage of the use of the energy in the increment as a Lyapunov function for control design is that a control law of the form (16) with  $Q_1 = Q$  can result in global stability of a more complex power system in which the original converter is embedded. In particular, if the converter is interconnected only with (relatively) passive circuit elements, the resulting interconnected system is always guaranteed to be stable. For example, if an additional section of output filter is added to the up-down converter of Fig. 2, as shown in Fig. 5, the control law designed for the original converter stabilizes the modified circuit.

#### D. Adaptive Control Method to Handle Uncertain Nominal State Values

This section considers a control design of the form (11) for the model (6), but in the case where the nominal state vector is unknown. The effect of this uncertainty is to replace the variable  $y$  by

$$y = (Bx + b)^*Q(x - \delta x_n)$$

where  $\delta x_n$  is the uncertainty in the nominal operating point, that is,

$$\delta x_n = \bar{x}_n - x_n \quad (20)$$

where  $\bar{x}_n$  is an estimate of the nominal operating point.

To implement the self-tuning scheme, we include as part of an augmented state vector, an estimate  $\bar{x}_n(t)$  of the constant nominal value of the state vector for the original plant. We can equivalently represent this estimate by its error, i.e.,  $\delta x_n(t) = \bar{x}_n(t) - x_n$ . The update law for  $\delta x_n(t)$  is selected by considering the Lyapunov function

$$V = \frac{1}{2}x^*Qx + \frac{1}{2}(\delta x_n)^*K(\delta x_n) \quad (21)$$

where  $K$  is a symmetric positive definite matrix and  $Q$  is as previously specified. In particular, it is possible to stabilize the system by choosing the update law to be

$$\frac{d}{dt}(\delta x_n) = -K^{-1}Q(Bx + b) \quad (22)$$

in conjunction with the control law (11). Note that  $y$  can now be determined without any uncertainty arising from the unknown nominal state values because

$$x - \delta x_n = (x_t - x_n) - (\bar{x}_n - x_n) = x_t - \bar{x}_n$$

where  $x_t$  is the actual full state value (which can be measured), whereas  $\bar{x}_n$  is stored in the controller. Note that it

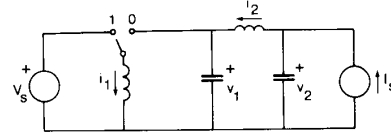


Fig. 5. Up-down converter with additional filter sections.

is generally possible to obtain an accurate measure of  $Q(Bx + b)$  as discussed in Section III.C.

*Example: Estimation of Nominal Inductor Current in Simple Up-Down Converter:* In this example, we apply the adaptive control scheme to the second order up-down converter whose parameters are given in Section III.A. We now assume, however, that the load is unknown but constant in the steady state. Consequently, the nominal inductor current is also unknown. This is the parameter that our self-tuning mechanism will estimate. In this example, it is assumed that the input voltage  $V_s$  is known (i.e., measured), the nominal output voltage  $v_n$  is defined by the regulation problem, and the nominal duty ratio  $d_n$  is known. (The nominal duty ratio can usually be determined from  $V_s$  and  $v_n$ .) We work with the augmented model

$$\begin{aligned} \begin{bmatrix} i' \\ v' \\ (\delta i_n)' \end{bmatrix} &= \begin{bmatrix} 0 & (1 - d_n)/L & 0 \\ -(1 - d_n)/C & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i \\ v \\ (\delta i_n) \end{bmatrix} \\ &+ \begin{bmatrix} (V_s - v_t)/L \\ i_t/C \\ -k^{-1}(V_s - v_t) \end{bmatrix} d \\ y &= (V_s - v_t) \{i - (\delta i_n)\} + i_t v. \end{aligned} \quad (23)$$

Note that in this model, the quantities without subscripts are deviations from nominal, the quantities with subscript  $t$  are total variables that can be measured, and the quantities with subscript  $n$  are nominal variables. We only attempt to estimate the nominal inductor current as the other nominal state variable (the capacitor voltage) is known. The output  $y$  of this model can be determined exactly since  $i - (\delta i_n)$  is precisely  $i_t - \bar{i}_n$ , i.e., the difference between the actual inductor current and the present estimate of the nominal value of this current. The control design can be completed by specifying  $k > 0$  and a feedback gain  $\alpha$ . These parameters may be selected by considering the small-signal behavior. For example, with a nominal load current of 2 A, the eigenvalues of the small-signal linearized model can be placed at  $-7.713 \pm j12.9$  and  $-11.36$  krad/s by selecting  $k = 2778$  and the unsaturated gain  $\alpha = 0.004$ . Of course, in practice, the design would have to be made without accurate knowledge of the nominal load current. Other parameter choices can result in still faster small-signal behavior. A numerical simulation of a start-up transient using these parameters is shown in Fig. 6. Note that the initial condition for the estimate of the nominal inductor was taken as zero.

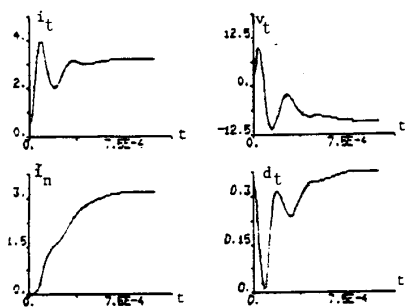


Fig. 6. Start-up transient in second-order converter using adaptive control scheme.

More complex examples that require estimation of more than one nominal state are considered in [23].

#### IV. SUMMARIZING REMARKS

The Lyapunov-based control described in this paper is evidently a promising approach to the control of switched-mode power converters. The method can be extended to converters that handle time-varying input-output waveforms, see the discussion in [23]. In applications in distributed power supply environments, this type of control may prove very useful as it may become necessary to stabilize arbitrary interconnections of converters and loads. The method also lends itself to the design of state observers as outlined in [26].

##### A. Passivity, Incremental Passivity, and Relative Passivity

In order to state the following definitions in a relatively general way, we assume the input-output vector pair of an  $n$ -port to be a *hybrid pair*. That is, the input  $u(t)$  and output  $y(t)$  of an  $n$ -port are  $n$ -component vectors whose elements represent port voltages or currents. The components of  $y(t)$  are complementary to those of  $u(t)$ , and oriented such that  $u(t)^*y(t)$  is the instantaneous power entering the network at its ports. In the following, the networks of interest are assumed to be time invariant unless otherwise noted.

*Passivity*: The definition of passivity presented in Wyatt *et al.* [17], Wyatt [19], and in Hasler and Neiryck [16] are adopted here.

*Definition A.1 (Available Energy)*: Given an  $n$ -port  $\mathbf{N}$ , let the available energy  $E_{A,x}$  in state  $x$  be the maximum energy that can be extracted from  $\mathbf{N}$  when its initial state is  $x$ , with the convention that  $E_{A,x} = +\infty$  if the available energy is unbounded. That is,

$$E_{A,x} = \sup_T \int_0^T -u(t)^*y(t) dt. \quad (24)$$

*Definition A.2 (Passivity)*:  $\mathbf{N}$  is passive if  $E_{A,x}$  is finite for each initial state  $x$ . Note that this definition of passivity is directly tied to a state-space realization for the  $n$ -port in question. This is not objectionable for our pur-

poses as we aim to draw conclusions for switching converters for which state models are readily obtained. In the context of a switching converter, the concept of passivity is of use in viewing a controlled converter as an interconnection of various  $n$ -ports.

*Incremental Passivity*: The definition given here follows the system theoretic framework of Desoer and Vidyasagar [20].

*Definition A.3 (Energy in the Increment)*: Given an  $n$ -port  $\mathbf{N}$  with initial state  $x$ , let  $(u_1(t), y_1(t))$  and  $(u_2(t), y_2(t))$  be any two admissible input-output trajectories on  $[0, T]$  with  $T$  finite. The energy in the increment between the two trajectories is defined by

$$W_x(T) = \int_0^T (u_1 - u_2)^*(y_1 - y_2) dt. \quad (25)$$

*Definition A.4 (Incremental Passivity)*: An  $n$ -port  $\mathbf{N}$  with initial state  $x$  is incrementally passive at state  $x$  if  $W_x(T)$ , the energy in the increment defined in (25), is nonnegative for every pair of admissible trajectories on  $[0, T]$  with  $T$  finite. If the network is incrementally passive at all states  $x$  in the state space, it is said to be incrementally passive. The  $n$ -port is strictly incrementally passive at state  $x$  if  $W_x(T) > 0$  whenever the two trajectories are distinct. The network is strictly incrementally passive if it is strictly incrementally passive at every state in the state-space. Note that this definition is closely tied to the definition of passivity. A passive network can supply only finite energy while an incrementally passive network can absorb only nonnegative energy in the increment between two trajectories ( $W_x$  in (25)).

*Relative Passivity*: Incremental passivity proves to be too strong a condition in the case of certain nonlinear  $n$  ports. In fact, many nonlinear networks that are not incrementally passive exhibit a closely related property that we term *relative passivity*. Another closely related notion, *local passivity* for a capacitor (or inductor), has been introduced in [16], [25]. However, our definition of relative passivity is potentially applicable to any type of network. To define a relatively passive network, we examine the energy in the increment with respect to a constant nominal operating point.

*Definition A.5 (Relative Passivity)*: Given an  $n$ -port  $\mathbf{N}$  with equilibrium state  $x_n$  and nominal output  $y_n$  corresponding to the constant input  $u_n$ , consider the admissible trajectory  $(u(t), y(t))$  on  $[0, T]$  that is obtained with initial state  $x(0) = x_n$ . The  $n$ -port is relatively passive at  $x_n$  if

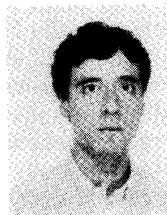
$$W_{x_n}(T) = \int_0^T [u(t) - u_n]^*[y(t) - y_n] dt \geq 0 \quad (26)$$

for any finite  $T$ . The  $n$ -port is relatively passive if (26) holds for any nominal operating point.  $\mathbf{N}$  is strictly relatively passive at  $x_n$  if the inequality in (26) is strict whenever  $x(T) \neq x_n$ .  $\mathbf{N}$  is strictly relatively passive if it is strictly relatively passive for any constant nominal state. In the case of lossless elements for which  $W_{x_n}(T)$  is a

function of only  $x_n$  and  $x(T)$ ,  $W_{x_n}(T)$  can be useful as a Lyapunov function.

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**Seth R. Sanders** received S.B. degrees in electrical engineering and physics from M.I.T., Cambridge, MA in 1981. He then worked as a design engineer at the Honeywell Test Instrument Division in Denver, CO. He returned to M.I.T. in 1983 and received the S.M. and Ph.D. degrees in electrical engineering in 1985 and 1989, respectively.

He is presently an assistant professor in the Department of Electrical Engineering and Computer Sciences at the University of California at Berkeley. His research interests are in the area of non-

linear control systems and circuits, and particularly in applications to power electronics, electromechanical systems, and power systems.



**George C. Verghese** received the B.Tech. degree from the Indian Institute of Technology, Madras, in 1974, the M.S. degree from the State University of New York, Stony Brook, in 1975, and the Ph.D. degree from Stanford University, CA, in 1979, all in electrical engineering.

He is Associate Professor of Electrical Engineering and a member of the Laboratory for Electromagnetic and Electronic Systems at the Massachusetts Institute of Technology, which he joined in 1979. His research interests are in the areas of systems, control, and estimation, especially as applied to power electronics, electrical machines, and bulk power systems.

Dr. Verghese is an Associate Editor of *Automatica* and of the IEEE TRANSACTIONS ON AUTOMATIC CONTROL. He is coauthor (with J. G. Kasakian and M. F. Schlecht) of *Principles of Power Electronics*, Addison-Wesley, City, 1991.