Parameter and State Estimation in Power Electronic Circuits *

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1 Introduction

This paper investigates techniques for reconstructing or estimating unmeasured circuit variables and parameters in power electronic circuits. In numerous applications in power electronic systems, measurements of circuit variables (currents and voltages) are required for closed-loop control purposes or for diagnostic purposes, but are expensive or impossible to obtain directly. Examples are certain inductor currents, voltages at high impedance nodes where probes cannot be applied, or circuit variables in electrically isolated portions of a circuit. The approach taken in this paper is based on observer theory [1, 2], and so the resulting algorithms may be implemented in real time. A natural scheme for implementing an observer is given here. Previous work on parameter and circuit waveform estimation [3] was based on off-line computational algorithms.

The paper is organized as follows. Section 2 briefly reviews observer theory, and introduces an approach for observer design for switching power circuits. The case considered in Section 2 is applicable only to the situation where circuit parameters are *known*. Since this is typically not the case in practice, Section 3 extends the method to the situation where the circuit contains unknown parameters. In particular, Section 3 develops an adaptive estimation algorithm for estimating unknown circuit parameters along with unmeasured circuit variables. An up-down converter is used as a vehicle throughout the paper to illustrate the results. Appendix A contains a discussion of the case where nonlinear resistive elements are present in the circuit.

2 Observer Theory - Application in Power Electronics

Consider the model of an isolated up-down converter of Figure 1 with state-space description

$$\dot{x} = Ax + (Bx + b)u + f \quad (1)$$
$$y = C(u)x + D(u) \quad (2)$$

where

$$A = \begin{bmatrix} 0 & N/L \\ -N/C & -1/R_0C \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -N/L \\ N/C & 0 \end{bmatrix},$$

$$b = \begin{bmatrix} V_g/L + NE_d/L \\ 0 \end{bmatrix}, \quad f = \begin{bmatrix} -NE_d/L \\ 0 \end{bmatrix},$$

$$C(u) = \begin{bmatrix} u & 0 \\ 0 & (1-u)N \end{bmatrix}, \quad D(u) = \begin{bmatrix} 0 \\ uV_g - (1-u)NE_d \end{bmatrix},$$

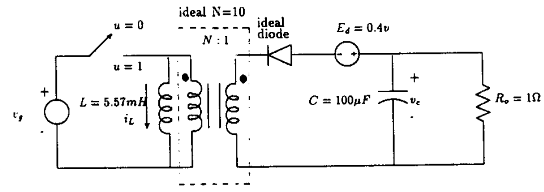

Figure 1: Model of an Isolated Up-Down Converter

x is the two-component state vector composed of the inductor current and the capacitor voltage, u takes on the value 0 or the value 1 depending upon the instantaneous switch position, and y contains measurements of the transformer primary current and voltage. Note that all dependence on time t is suppressed in the notation. In the case where all parameter values are known, the first step in constructing a state observer for the system (1) is to construct a system that exactly copies the dynamics of this system. Such a system would take the form

$$\dot{z} = Az + (Bz + b)u + f \quad (3)$$

where the vector z is an estimate of the state vector x. It is essential that the observer state z asymptotically converges to the underlying system state x. In order to study this behavior, one needs to examine the error dynamics that govern the error $e = z - x$, i.e.,

$$\dot{e} = Ae + uBe \ . \quad (4)$$

Since this dynamics may not in general be guaranteed to be stable or may not result in optimal estimates, observer theory leads one to incorporate a prediction error term formed from the difference between the measurement y of the output in (2) and the predicted output $C(u)z + D(u)$. The observer is then completed by injecting a signal proportional to the prediction error into the right-hand side of (3) yielding the observer system

$$\dot{z} = Az + (Bz + b)u + f + K(t)[C(u)z + D(u) - y] \quad (5)$$

and the associated error dynamics

$$\dot{e} = Ae + uBe + K(t)C(u)e. \quad (6)$$

A standard result of linear system theory [1, 2] guarantees that one can find a $K(t)$ that stabilizes this dynamics provided the system modeled by (1) and (2) is *observable*.

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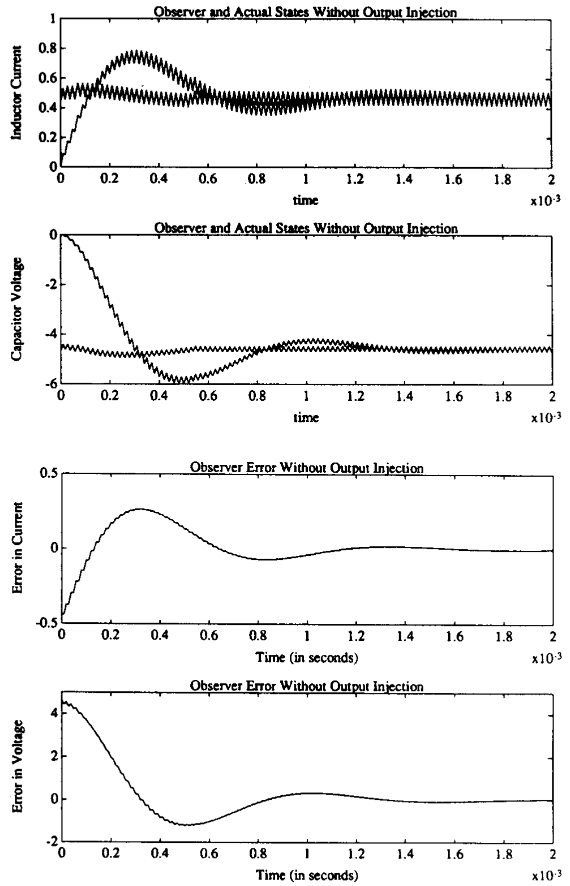

Figure 2: Simulation of Open Loop Observer: (a) Actual and Observer Currents, (b) Actual and Observer Voltages, (c) Observer Current Error, and (d) Observer Voltage Error

It turns out that the open loop error dynamics (4) is asymptotically stable for the example of Figure 1. Futhermore, there is a natural Lyapunov function for this error dynamics given by

$$V(e) = \frac{1}{2}e^*Qe \tag{7}$$

where $Q = diag\{L\, C\}$ and * indicates the transpose of the associated vector. This function corresponds to the ***energy in the increment*** between the trajectories of the observer system and the trajectories of the underlying boost converter. See [4, 5] for more details on the nature of this Lyapunov function and its use in exhibiting open loop stability of switching power converters. Results in [4, 5] guarantee that the ***energy in the increment*** is a Lyapunov function for a power electronic circuit that is built from linear passive reactive elements, ideal switches, time-varying sources, and incrementally passive resistive elements. One conclusion for such a circuit is that any pair of trajectories corresponding to differing initial conditions cannot diverge. Since the open loop observer and the underlying converter system correspond to identical dynamical systems that may be initialized with different initial conditions, we obtain

$$\dot{V}(e) = \frac{1}{2}\{e^*[QA + A^*Q]e + ue^*[QB + B^*Q]e\} \leq 0. \tag{8}$$

We conclude that the observer error (measured by $V(e)$) cannot increase. A simulation of this open loop observer has been carried out for the example up-down converter of Figure 1. The results are shown in Figure 2. Note that the convergence of the error can be slow, and is actually controlled by the open loop circuit dynamics.

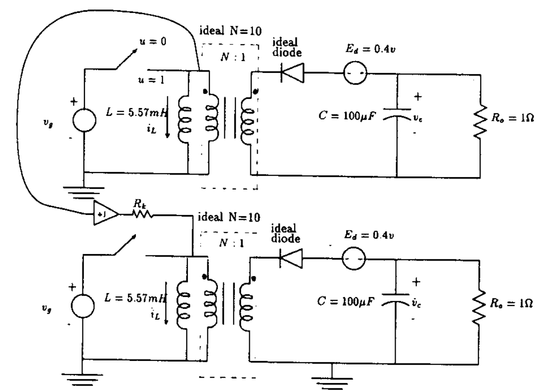

Figure 3: Observer Implementation Scheme

It is possible to improve upon the open loop observer by incorporating a prediction error gain of the form $K(u) = -Q^{-1}C^*(u)R(u)$ where $R(u)$ is a positive semi-definite matrix for $u = 0, 1$. This gain results in the error dynamics

$$\dot{e} = Ae + uBe - Q^{-1}C^*(u)R(u)C(u)e. \tag{9}$$

Differentiating the Lyapunov function (7) along this error dynamics yields

$$\begin{aligned} \dot{V}(e) &= \frac{1}{2}\{e^*[QA + A^*Q]e + ue^*[QB + B^*Q]e + \\ & \quad -2e^*C^*(u)R(u)C(u)e\} \ \leq 0. \end{aligned} \tag{10}$$

In practice, we could select $R(u)$ to yield relatively fast *averaged* error dynamics controlled by the eigenvalues of $A + d_nB - Q^{-1}\overline{C^*RC}$ where d_n is the nominal (constant) duty ratio and $\overline{C^*RC}$ is the averaged value of the matrix $C^*(u)R(u)C(u)$. Other criteria for the selection of R could be based on minimization of steady state errors due to parameter uncertainty or due to noise entering the system dynamics and the measurement equations [6]. Note that the observer with this form of prediction error gain can be implemented in a natural way as illustrated in Figure 3 for the example up-down converter. The circuit corresponds to a copy of the up-down converter with the prediction error injection implemented by a single resistor. For simplicity in this example, we have only used the measurement of primary transformer voltage which corresponds to a singular matrix R. Note that all impedances could be scaled so that the observer has reduced currents, compared to those of the underlying circuit. A simulation of waveforms obtained with this observer is shown in

Figure 4. Note that the errors converge more rapidly towards zero.

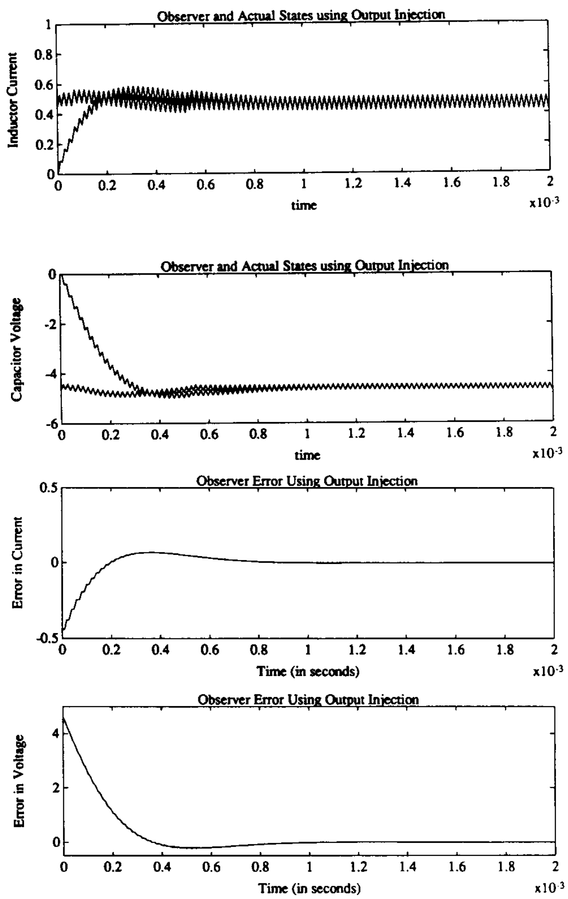

Figure 4: Simulation of Observer Using Prediction Error: (a) Actual and Observer Currents, (b) Actual and Observer Voltages, (c) Observer Current Error, and (d) Observer Voltage Error

It turns out that the approach taken in this section for constructing observers can be extended to the case where nonlinear resistive elements are present in the converter circuit. The only requirement is that these resistive elements be incrementally passive. A nontrivial example of a power electronic circuit with a such nonlinear resistive element is a DC-DC converter operating in the discontinuous conduction mode. Appendix A gives details on this case.

3 Unknown Circuit Parameters

The development in the previous section assumed that all circuit parameters were known a priori. To study the effects of unknown parameter values, the observer was simulated when the load resistance was perturbed by 5%. The results are shown in Figure 5. One of the most notable features of this simulation is that there is a steady state bias in the state estimates produced by the observer.

A natural approach for dealing with this problem is to construct an adaptive observer that estimates the unknown circuit parameters along with the circuit state variables. In the sequel,

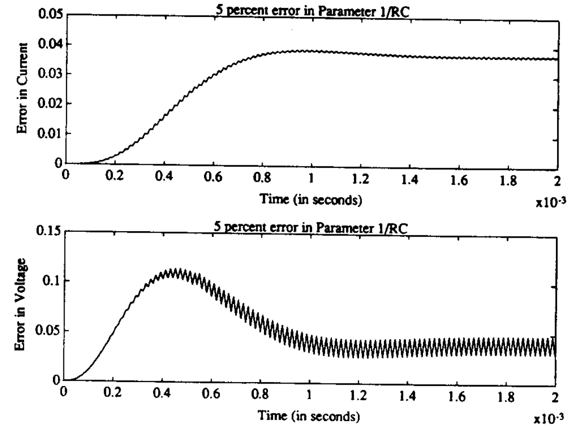

Figure 5: Observer Waveforms with Parameter Uncertainty: (a) Observer Current Error and (b) Observer Voltage Error

we shall assume that the switch variable u is known for all time, and that the portion of the measurement equation (2) that involves the state is exact. That is, the matrix $C(u)$ is known. However, we shall relax the assumption that the circuit parameters appearing in the state-space model (1) (and in $D(u)$) are known. For the purposes here, define

$$A(u) = A + uB \qquad (11)$$

$$b(u) = bu + f \qquad (12)$$

so that the model (1) can be rewritten in the form

$$\dot{x} = A(u)x + b(u). \qquad (13)$$

Since the parameters of this model are not precisely known, a first step in constructing an observer would be to use the best known estimates of the unknown parameters. The observer system corresponding to (5) could then take the form

$$\dot{z} = \hat{A}(u)z + \hat{b}(u) \qquad (14)$$

where $\hat{A}(u)$ and $\hat{b}(u)$ are the available estimates of $A(u)$ and $b(u)$, respectively. We could introduce an error injection term to speed up convergence, but this is not done to keep the presentation simple. (Note that the measurements will be used to update estimates for the parameters in the system model.) The observer error $e = z - x$ is now governed by

$$\dot{e} = A(u)e + \delta A(u)z + \delta b(u) \qquad (15)$$

where $\delta A(u) = \hat{A}(u) - A(u)$ and $\delta b(u) = \hat{b}(u) - b(u)$. It is generally possible to parameterize the circuit in a manner such that

$$A(u)x + b(u) = W(x,u)\theta \qquad (16)$$

where $W(x,u)$ is a known matrix (depending on x and u) and all the unknown parameter information is contained in the vector θ. The important feature of this choice is that it is a *linear* parameterization. With this parameterization, the error equation (15) becomes

$$\dot{e} = A(u)e + W(z,u)\delta\theta \qquad (17)$$

where $\delta\theta = \hat{\theta} - \theta$ is the vector of parameter errors. The solution of (15) is given by

$$e(t) = \Phi(t,t_0)e(t_0) + \int_{t_0}^{t} \Phi(t,\tau)W(z(\tau),u(\tau))\delta\theta d\tau . \qquad (18)$$

where $\Phi(t, t_0)$ is the state transition matrix corresponding to the system matrix $A(u)$. The corresponding error in the output $\delta y = C(u)(z - x) + \delta D(u)$ satisfies

$$\delta y = C(u)[\Phi(t,t_0)e(t_0) + \int_{t_0}^{t} \Phi(t,\tau)W(z(\tau),u(\tau))\delta\theta d\tau\ ] + \delta D(u). \tag{19}$$

The vector $D(u)$ can also be expressed in the form $W_1(u)\theta$, where $W_1(u)$ is known. One then finds

$$\delta y = C(u)\Phi(t,t_0)e(t_0) + H(t)\delta\theta \tag{20}$$

where $H(t) = C(u)\int_{t_0}^{t} \Phi(t,\tau)W(z(\tau),u(\tau))d\tau + W_1(u)$, provided $\delta\theta$ is constant.

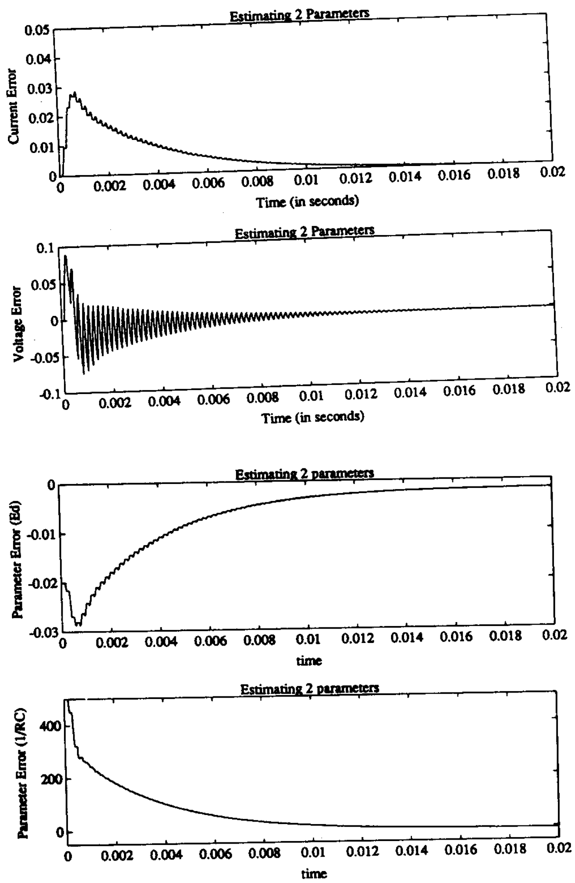

Figure 6: Adaptive Observer Waveforms: (a) Error in Current Estimate, (b) Error in Voltage Estimate, (c) Error in Estimate of E_d, and (d) Error in Estimate of R_0

Noting that the term $\Phi(t,t_0)e(t_0)$ asymptotically decays to zero, we are led to attempt a parameter update law based on the gradient algorithm [7]. Such an algorithm takes the form

$$\dot\theta = -gH^*\delta y \tag{21}$$

where g is a gain parameter. In practice, the system matrix $A(u)$ is unknown, so we instead use the available estimate of this matrix $\hat{A}(u)$ to construct an estimate $\hat{H}(t)$ of $H(t)$ via

$$\hat{H}(t) = C(u)\int_{t_0}^{t} \hat{\Phi}(t,\tau)W(z(\tau),u(\tau))d\tau + W_1(u) \tag{22}$$

where $\hat{\Phi}(t,\tau)$ is the transition matrix that corresponds to the system matrix $\hat{A}(u)$. Note that $\hat{H}(t)$ converges to $H(t)$ if the unknown system parameters converge to their actual values. The error in the parameter vector is then governed by the approximate dynamics

$$\frac{d}{dt}\delta\theta \approx -g\,\hat{H}(t)^*H(t)\,\delta\theta \tag{23}$$

for slowly varying $\delta\theta$. For this analysis to hold, the gain g needs to be selected so that the parameter update dynamics (23) evolve much more slowly than the observer error dynamics.

The convergence of the parameter error system can be studied via averaging analysis [7]. In particular, for a sufficiently small gain g, the error dynamics are known to be exponentially stable if the *averaged* error dynamics are exponentially stable. One can therefore consider the stability properties of the averaged dynamics

$$\frac{d}{dt}\delta\theta \approx -g\,\overline{\hat{H}(t)^*H(t)}\,\delta\theta \tag{24}$$

to assess the stability of the parameter error system. The convergence of the averaged parameter error dynamics can be studied with a Lyapunov function of the form $V(\delta\theta) = (1/2)(\delta\theta)^*(\delta\theta)$. Differentiating along the averaged parameter error system trajectories, we obtain

$$\dot{V}(\delta\theta) = -g\,(\delta\theta)^*\,\overline{H(t)^*H(t)}\,(\delta\theta) - g\,(\delta\theta)\,\overline{\delta H(t)^*H(t)}\,(\delta\theta) \tag{25}$$

where $\delta H(t) = \hat{H}(t) - H(t)$. The right-hand side of this expression is nonpositive as long as the matrix norm of $\overline{\delta H(t)^*H(t)}$ does not exceed the smallest eigenvalue of $\overline{H(t)^*H(t)}$. This is a rather conservative condition for stability, but the important point is that there is some neighborhood of convergence. One interesting point is that it is possible to estimate $\overline{H(t)^*H(t)}$ off-line using a nominal steady state trajectory to assess the feasibility of estimating certain parameters given certain measurements. Typical analyses show that it is always feasible to estimate a single parameter, and sometimes feasible to estimate pairs of parameters.

Here, we illustrate the algorithm on our example up-down converter. For purposes of illustration, we assume that all parameters, except the load resistance and the diode forward voltage drop, are known, and hence the adaptive observer is designed to estimate these parameters along with the circuit state variables. The results of a simulation are shown in Figure 6. This simulation used initial parameter values (load resistance and forward diode drop) that had five percent errors. Note that in this example, the estimator would allow one to avoid all measurements of secondary side circuit variables.

4 Conclusion

This paper has explored techniques for estimating power electronic circuit waveforms and parameters using only a limited set of measurements. The observer scheme outlined in Section 2 admits a simple circuit implementation. We aim to develop a similar implementation scheme for the adaptive estimator introduced in Section 3. With this approach, one will be able to avoid difficult or expensive circuit measurements

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A The Case of Nonlinear Resistive Elements

In the case where nonlinear resistive elements are present in a power circuit, the state-space model can be recast as

$$Q\dot{x} = -(1-u)\mathcal{H}_0(x) - u\mathcal{H}_1(x) \quad (26)$$
$$y = C(u)x + D(u) \quad (27)$$

where x is the vector of inductor currents and capacitor voltages, Q is the matrix of inductances and capacitances, and $\mathcal{H}_i(\bullet)$ are hybrid representations for the non-reactive portion of the circuit that correspond to each of the switch configurations. A consequence of the incremental passivity of the resistive elements is that the hybrid representations are also incrementally passive, i.e.

$$(z-x)^*[\mathcal{H}_i(z) - \mathcal{H}_i(x)] \geq 0 \quad (28)$$

for any x and z and for $i = 0, 1$. An observer for the system modeled by (26,27) can then take the form:

$$Q\dot{z} = -(1-u)\mathcal{H}_0(z) - u\mathcal{H}_1(z) + K(u)[C(u)z + D(u) - y]. \quad (29)$$

A choice of $K(u) = C(u)^*R(u)$ with $R(u)$ positive semi-definite will then result in stable error dynamics, obtained by subtracting (26) from (29). This can be demonstrated by differentiating the the energy in the increment $V = \frac{1}{2}(x-z)^*Q(x-z)$ along trajectories of the error system. Carrying out this procedure, one finds

$$\begin{aligned}\dot{V} = & -(1-u)(z-x)^*[\mathcal{H}_0(z) - \mathcal{H}_0(x)] \\ & -u(z-x)^*[\mathcal{H}_1(z) - \mathcal{H}_1(x)] \\ & -(z-x)^*C(u)^*R(u)C(u)(z-x). \end{aligned} \quad (30)$$

The first two terms on the right-hand side of (30) are nonpositive as a result of the incremental passivity of the resistive elements in the circuit, while the third term is nonpositive by design. The conclusion is that the error dynamics are stable. Asymptotic stability can typically be obtained in practice. A nontrivial example of a circuit containing such a nonlinear resistive element is a converter operating in discontinuous conduction mode. Figure 7 shows a model of an up-down converter containing an ideal single-pole double-throw switch and an ideal diode that prevents the inductor current from reversing. In the discontinuous conduction mode, the diode can be viewed as a nonlinear incrementally passive resistive element. As such, it is possible to construct an observer for this circuit using the framework of this paper.

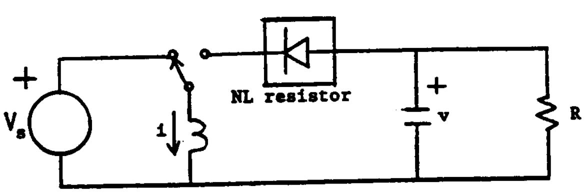

Figure 7: Up-Down Converter Model for Discontinuous Conduction Mode

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