RELIABILITY ANALYSIS VIA NUMERICAL SIMULATION OF POWER ELECTRONIC CIRCUITS

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ABSTRACT

This paper gives conceptual background on reliability analysis in the presence of circuit parameter variation. A method for reliability analysis, the first-order reliability method (FORM) [1], is described in detail and demonstrated on circuit examples. Simulation results are then compared to a Monte Carlo analysis.

1 INTRODUCTION

Recent reductions in computation time for power circuit simulation make it practical to use simulation to predict weaknesses in power circuit designs [2] [3] [4] [5]. A logical next step is to incorporate simulation as a tool for analyzing the reliability of a given design in the presence of parameter variations in manufacturing. Ultimately, this *analysis* is a step in *reliability design.* (See Figure 1.) The integrated circuit computer-aided design (IC CAD) literature [6] [7] and structural reliability literature [1] offer a wealth of ideas addressing the need to incorporate parameter variations into the design process. In IC manufacturing, however, several parameter variations can be correlated due to their being created in the same process, significantly reducing the number of actual parameters to be considered. Thus, the difference in focus in methods that must be considered in discrete circuitry, such as that used in power electronics, is that the approach must be generalizable to circuits with high dimensional parameter spaces.

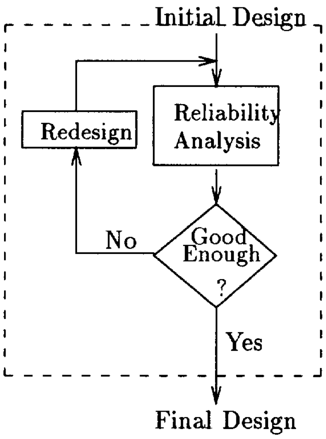

Figure 1: Reliability Design

Note that a certain initial investment is needed for analysis and determination of an optimal design, but this can amount to a big savings in mass production. The remainder of the paper will concentrate on analysis, the inner loop of the design process.

2 BACKGROUND

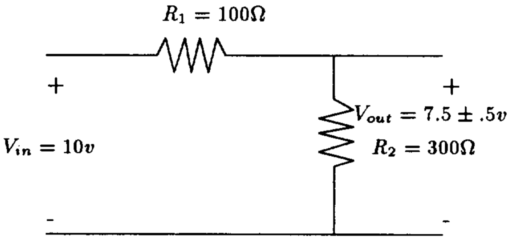

Figure 2: Voltage Divider

If there are n variable parameters in a circuit, then an n-vector of parameter values (a point in the parameter space) can be used to represent that circuit at those values. Typical parameters correspond to component values, including, but not limited to, resistances, capacitances, inductances, and semiconductor parameters. Statistical variations of component values result in parameter variation, possibly centered around the ideal values (design center) of each component.

Usually a lengthy set of simulations is required to determine the functionality of a circuit at a particular parameter vector. For example, simulating a circuit might require a calculation of its steady-state waveforms, the data required to judge circuit functionality. Because of lengthy simulations and because many iterations of analysis may be required to perform the reliability analysis, an effective procedure should conserve simulations.

We will use a simple two-component circuit example, the voltage divider of Figure 2, to illustrate reliability concepts and terminology. Consider the analysis of the voltage divider with the two resistors as the allowable design parameters. The design goals are to have an open-circuit output voltage, V_o, at 7.5 ± .5 Volts and output impedence, Z_o, at $75 \pm 25\Omega$. The nominal parameter values at which $V_o = 7.5V$ and $Z_o = 75\Omega$ are $R_1 = 100\Omega$ and $R_2 = 300\Omega$.

2.1 Terminology

Performance Function. In the above example, the *performances* are output voltage, V_o, and output impedence, Z_o. These are the outputs of interest. The bounding curves are based on a performance function specification (ex: "Output voltage shall be between 7 and 8 Volts."). In a more general problem, there will not be access to the performance function equations explicitly. This is explained further in Section 2.2.

Region of Acceptability. The *region of acceptability*, R_A, is the set of vectors in the space of circuit parameters that results in a circuit whose *performance functions* are deemed acceptable. This is shown in Figure 3. The equations of the curves bounding R_A are

$$V_o(R_1, R_2) = \frac{R_2}{R_1 + R_2} V_{in} = \begin{cases} 7V \\ 8V \end{cases} \text{ and } \quad (1)$$

$$Z_o(R_1, R_2) = \frac{R_1 R_2}{R_1 + R_2} = \begin{cases} 50\Omega \\ 100\Omega \end{cases} . \quad (2)$$

The bounding curves in Figure 3 were drawn because we have access to the equations defining V_o and Z_o as a function of R_1 and R_2. These curves are the boundaries at which one or more performance criteria are violated. The intersection of the regions between the two pairs of bounding curves is exactly R_A. The area outside R_A, weighted by the probability density functions of the parameters, is the probability of failure: $P_f = 1 - \int_{R_A} f_x(x)dx$. Equivalently, the area inside that region, weighted by the probability density functions of the parameters is the yield: $1 - P_f = \int_{R_A} f_x(x)dx$=yield.

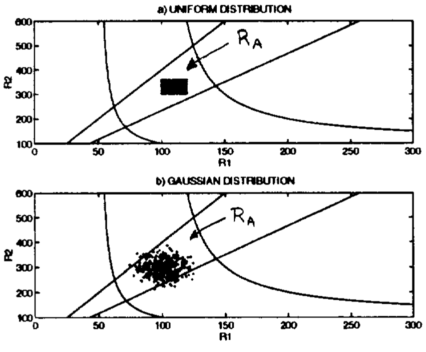

Figure 3: Region of Acceptability with distribution a) $f_x(x)$ =uniform, and b) $f_x(x)$ =Gaussian.

2.2 Background of Approaches to Reliability Analysis

A complicating feature of some circuits is that the performance functions ($V_o(R_1, R_2)$ and $Z_o(R_1, R_2)$ in the above example) are not simple equations, but rather the implicit solutions to difficult algebraic or differential algebraic equations. This makes determining R_A non-trivial. Thus, a series of structured simulations is needed to approximate R_A.

Given the use of simulation, there is some choice as to which simulations should be done to determine yield. One easily implementable method is a Monte Carlo analysis [8]. However, for very low failure rates, a Monte Carlo analysis requires a very large number of simulations. For example, consider a design that aspires to six sigma [9][10] standards (about 4 failures per million). If we want to have 95% confidence that the percent error on a yield estimation is 100% or less for an estimated probability of failure, $P_f = 4 \times 10^{-6}$, then approximately 10^6 simulations will be necessary. (See Appendix A for details.) The high number of required simulations is due to the fact that it takes many simulations to get a failure when failure rates are low. Also note that a Monte Carlo selection does not incorporate information from past simulations to influence the choice of future simulations. There is no structured plan other than to emulate the assumed statistics in the selection of parameter vectors to simulate. However, there is work to circumvent these difficulties with the Monte Carlo approach, see [11] for example.

Structured simulation approaches include sensitivity analyses where one parameter at a time is perturbed to see the effects on performance. These have advantages and provide useful information, but only provide small signal effects. Obtaining large signal effects would require simulating points throughout the entire parameter space, a task that increases rapidly with parameter space dimensionality. Other structured simulation approaches are described in [6]. For various reasons, including the shortcomings of methods described above, the approach to reliability analysis pursued in this paper is a first-order reliability method, described in the next section.

3 FIRST-ORDER RELIABILITY METHOD (FORM)

Given a design center and parameter distributions, FORM [1] methodically conducts simulations to search for the statistically most likely point of failure for a particular performance specification (e.g. $V_o \leq 8V$ in Equation 1). It then determines the tangent hyperplane along the boundary of R_A at the most likely point of failure. The yield is computed based on that first-order model, which divides the parameter space into pass/fail half-spaces. The calculations are done by first transforming the distribution space to the standard normal space, which facilitates the yield integration. When several performance criteria are specified, the R_A approximation is a logical function of the half-spaces. The FORM algorithm is implemented here by CALREL [12] software, which was developed to analyze structural reliability, but which is applicable to the reliability of a design under parameter variation. Whereas structural reliability emphasizes the determination of P_f of one structure, manufacturability emphasizes the determination of P_f of a series of structures.

The underlying assumptions are 1) simulations are accurate; 2) simulations dominate time; and 3) each output performance function is continuously differentiable over the parameter space. This last assumption is needed to numerically calculate a gradient when one is not available analytically.

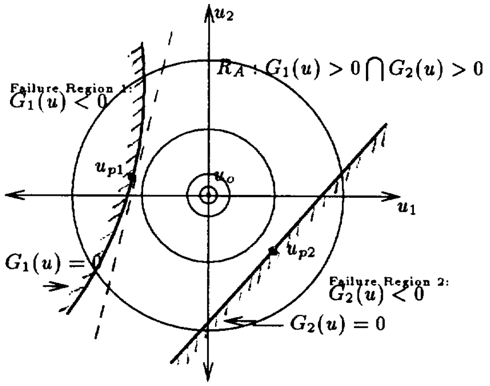

Figure 4: Determination of most likely points of failure, u_{p1} and u_{p2}, in the standard normal space.

3.1 FORM Algorithm

The algorithm is as follows. Refer to Figure 4 to illustrate these steps.

1. Define limit-state functions such that $g_i(x) \leq 0$ implies failure and $g_i(x) > 0$ implies a passing circuit. This is done for all $i = 1, 2, 3, \ldots, n_p$, where n_p = the number of performance specifications that must be satisfied, x = the parameter vector.
2. Incorporate component statistics. Transform variables to a set of uncorrelated standard normal variables: $u = \Phi^{-1}(x)$. Figure 4 shows two limit-state function surfaces, $G_1(u) = 0$ and $G_2(u) = 0$, in the transformed space.
3. Determine the point of most likely failure for each of the n_p specifications. This is a standard optimization problem under some assumptions.
4. Compute the tangent hyperplanes.
5. Perform logic to half-spaces and determine P_f.

3.2 Implementation

The software implementation is described by the block diagram of Figure 5. Statistical information is given to the program, CALREL conducts the FORM algorithm using calls to the circuit simulator. The program concludes when most-likely failure points, reliabilities, yields, and the sensitivities of those values are computed. Execution of the analysis required writing scripts to allow communication between programs. It also required making adjustments to the optimization step size and convergence tolerance. The optimization of step 3 in the FORM algorithm above requires a gradient calculation. Since performance as a function of parameters is not explicitly known, a gradient is calculated by the finite-difference method. This is where an assumption that each performance function be a continuously differentiable function of parameters is neeeded. This aspect needs to be explored more fully to understand in which cases this algorithm can or cannot be expected to work. One advantage of Monte Carlo methods is that the differentiability assumption is not neccesary. However, this is at the expense of needing a high number of simulations when P_f is low.

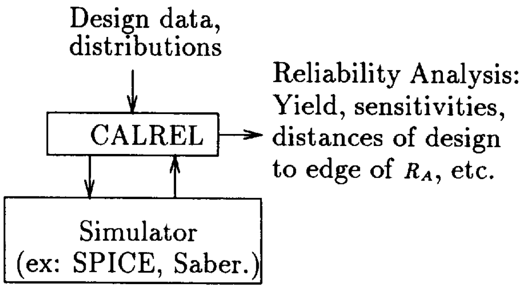

Figure 5: Data Flow

3.3 Application of FORM to Voltage Divider

For the voltage divider example, the parameter vector is $x = (R_1, R_2)$. Responses to various parameter vectors are computed by calling SPICE [13] from CALREL [12]. The gradient is calculated by a finite difference method. Since simulation times can be long in general, reduced simulation counts will be a criterion for usefulness of this method and are listed in Table 1.

Step one results in the limit-state functions:

$$
\begin{aligned}
g_1(R_1, R_2) &= V_o(R_1, R_2) - 7 && (3)\\
g_2(R_1, R_2) &= 8 - V_o(R_1, R_2) && (4)\\
g_3(R_1, R_2) &= Z_o(R_1, R_2) - 50 && (5)\\
g_4(R_1, R_2) &= 100 - Z_o(R_1, R_2). && (6)
\end{aligned}
$$

Step two, assuming a jointly Gaussian distribution of parameters, performs the following affine transformation to the standard normal form:

$$u = \Sigma^{-\frac{1}{2}}(x - M), \qquad (7)$$

where M =mean vector, $\Sigma = \Sigma^{\frac{1}{2}}\Sigma^{\frac{1}{2}T}$= covariance matrix.

In step three, the most-likely failure points for each limit-state function are shown in Figure 6. Tangent planes at those points and the appropriate calculation results in $P_f = .06248$ after 373 simulations. A Monte Carlo analysis results in a similar probability of failure, $P_f = .06$, after 1000 simulations.

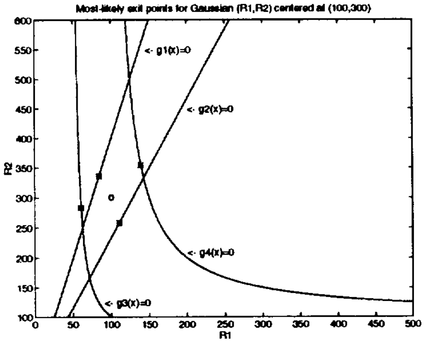

Figure 6: Voltage Divider run 1.

3.4 Application of FORM to Rectifier

In addition to a voltage divider, FORM was applied to the rectifier of Figure 7. The resistors were varied in the circuit, assuming a Gaussian distribution and only one limit-state function, $g_1(R_1, R_2) = V_o(R_1, R_2) - 17.5V$. Thus, the number of simulations was lower than for the voltage divider example. Since the probability of failure is much lower for this example, the potential savings over Monte Carlo analysis is greater. A total of 85 simulations were required in the FORM calculation. According to Appendix A, to get a 10% error in a Monte Carlo analysis with $P_f = .002$ would require about *200,000* simulations.

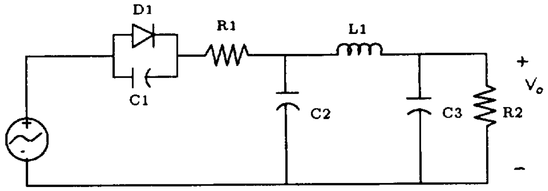

Figure 7: Rectifier

Table 1 summarizes the reliability analyses of the above circuits.

4 CONCLUSIONS/DIRECTIONS OF FUTURE RESEARCH

Simulation and concepts from manufacturing and structural reliability can be used as tools for power electronic circuit analysis. In particular, the first-order reliability method (FORM), in use in structural reliability, was shown to give a good estimate of P_f with a large savings in simulations compared to a Monte Carlo analysis when failure probabilities are very small. These concepts were illustrated on a voltage divider and a rectifier.

Future work includes conducting experiments with more complicated circuits, including compensated circuits, and studying the differentiability of performance functions over the parameter space. When the analysis implementation on more complicated circuits becomes better understood, the next stage of work would be to "close the loop" of Figure 1 and incorporate the analysis into a design optimization scheme.

Appendix A. There is a 95% chance that the percent error in the estimated probability will be less than that given by

$$\%error = 200\sqrt{\frac{1 - P_f}{nP_f}}, \qquad (8)$$

where P_f= estimated probability of failure and n= required number of simulations [14].

REFERENCES

[1] H. Madsen, S. Krenk, and N. Lind, *Methods of Structural Safety*. Prentice-Hall, Inc., 1986.

[2] E. Lee and T. Wilson Jr., "Methodology for using circuit simulation in the design and development of electronic power supplies," *IEEE Power Electronics Specialists Conference*, pp. 34–45, 1992.

[3] R. Michelet and W. Roeher, "Evaluating power supply designs with cae models," *IEEE Applied Power Electronics Conference Proceedings*, pp. 323–334, 1989.

Run	Experiment	FORM: P_f, no. simulations	Monte-Carlo: $E(P_f)$, no. simus, %error
1	Voltage Divider $\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} \sim N\left(\begin{bmatrix} 100 \\ 300 \end{bmatrix}, \begin{bmatrix} 10 & 0 \\ 0 & 30 \end{bmatrix}\right)$	$P_f = .06248$ 373	$E(P_f) = .06$ 1000, 3.3%
2	Voltage Divider $\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} \sim N\left(\begin{bmatrix} 100 \\ 300 \end{bmatrix}, \begin{bmatrix} 10 & 0.2 \\ 0.2 & 30 \end{bmatrix}\right)$	$P_f = .0384$ 610	$E(P_f) = .044$ 1000, 4.3%
3	Rectifier $\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} \sim N\left(\begin{bmatrix} 100 \\ 5.0 \end{bmatrix}, \begin{bmatrix} 100 & 0 \\ 0 & 0.5 \end{bmatrix}\right)$	$P_f = .001967$ 85	$E(P_f) = .002857$ 1400, 68.8%

Table 1: Numerical Results.% error in column 4 refers to error with 95% confidence. See Appendix A.

[4] Power Design Tools, Inc., *Simplis: A SIMulator for Piecewise-Linear Systems.*

[5] Analogy, Inc., *Power Express: Putting Power Electronics Design on the Fast Track with SABER.*

[6] R. Spence and R. Singh Soin, *Tolerance Design of Electronic Circuits.* Addison-Wesley, 1988.

[7] S. Director, P. Feldmann, and K. Krishna, "Stastical integrated circuit design," *IEEE Journal of Solid-State Circuits*, pp. 193–202, March, 1993.

[8] J. Hammersley and D. Handscomb, *Monte Carlo Methods.* Chapman and Hall, N.Y., 1983.

[9] R. White, "An introduction to six sigma with a design example," *IEEE APEC*, pp. 28–35, 1992.

[10] R. White, "Component tolerance and circuit performance: A case study," *IEEE APEC*, pp. 922–927, 1993.

[11] D. Cameron, J. Lang, and D. Belanger, "The computer-aided design of variable-reluctance generators," *IEEE APEC*, pp. 114–120, 1992.

[12] P. Liu, H. Lin, and A. Der Kiureghian, *CALREL User Manual, Report No. UCB/SEMM-89/18.* Dept. of Structural Engineering, Mechanics and Materials, UC Berkeley, August, 1989.

[13] B. Johnson, T. Quarles, and etal., *SPICE3 Version 3f User's Manual.* Dept. of EECS, UC Berkeley, October 1992.

[14] M. Shooman, *Probabilistic Reliability: An Engineering Approach.* McGraw-Hill Book Co., New York, N.Y., 1968.