Fundamental Limits on Energy Transfer and Circuit Considerations for Piezoelectric Transformers

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Abstract

This work investigates fundamental limits on electromechanical energy conversion capacity of piezoelectric transformers by considering a work cycle analysis. Fundamental limitations are imposed by maximum electric field strength, maximum surface charge density, maximum stress, and maximum strain. Our analysis indicates that the mechanical stress limit is the effective constraint in typical PZT materials. For a specific PZT-5H sample considered, a mechanical stress-limited work cycle indicates that this material can handle 330W/cm^3 at 100 kHz. The theory of operation of soft-switching resonant drive circuitry is then discussed. Experimental results on a soft-switching inverter incorporating *no magnetic* components are reported.

I Introduction

Power electronic circuits have conventionally been based on magnetic technology and until recently have not been part of the tide of miniaturization and integration advances from which signal-processing integrated circuits have benefited. In many power electronic systems today, the magnetic components remain as the bulkiest and costliest components. In an effort to achieve miniaturization, we are investigating alternative technologies to magnetics for provision of reactance, and voltage/current scaling (transformer function). Specifically, we are investigating acoustic coupling in place of inductive coupling to create electro-mechanical transformers.

Acoustic coupling can be achieved through piezoelectric or magnetostrictive means. A piezoelectric transformer offers the potential advantages of a thinner profile than a magnetic transformer, and a simpler manufacturing process due to the lack of windings. Application areas range from battery-powered consumer electronics and cellular telephones to wearable computers and head-mounted displays. The driving application at present for a piezoelectric transformer is the step-up transformation required to power a cold cathode fluorescent lamp in a flat panel display from a laptop computer battery. These cold cathode fluorescent lamps require 1000 V to start and 400-500 V to run at a 2-4 W power level and have an input impedance on the order of hundreds of kilo-ohms, a good match for the piezoelectric transformer.

While piezoelectric transformers were originally developed in the United States in the 1950s [1,2], they were not commercially pursued to a large extent due to poor materials reliability and competition from magnetic flyback transformers for cathode ray tubes. Recently, a number of researchers, mainly in Japan, have begun new efforts at producing compact, efficient, bulk-ceramic piezoelectric transformers for portable applications, most notably for laptop flat panel displays. These efforts have yielded reported efficiencies from 82% [3] to as high as 92% [4].

In this paper, we discuss fundamental limits of energy density and power throughput of a piezoelectric transformer based on materials considerations and work cycles. This analysis leads to the possibility of soft-switching and we describe an inverter circuit that we have built which achieves

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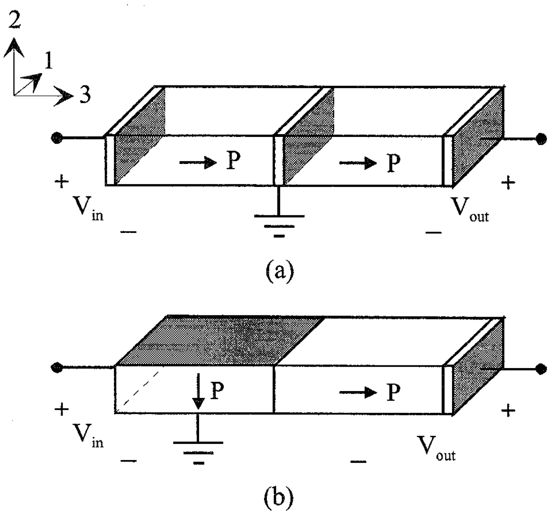

(a)

(b)

Figure 1. Both transformers shown here are long thin bars of piezoelectric material driven in a longitudinal mode resonance (displacements in the 3-direction) [1]. (a) Two segments are poled as shown and electroded faces perpendicular to the poling directions provide contacts for the input and output voltages. (b) The input segment is poled transversely to the longitudinal mode displacements, utilizing the d_{31} effect (d_{31}=d_{32}). The generator segment uses d_{33} coupling, a larger effect than d_{31}, which assists in creating larger voltage amplification.

soft-switching and high efficiency without the use of magnetic components.

II Electromechanical Power Transformation – Fundamental Limits

A piezoelectric transformer works by using the direct and converse piezoelectric effects to acoustically transform power from one voltage and current level to another. That is, power is converted electromechanically through a vibrating piezoelectric structure rather than electromagnetically as in an inductive transformer. Rosen first described piezoelectric transformers in [1,2], and two of the simplest structures are shown in Figure 1.

In the transformers shown in Figure 1, the converse piezoelectric effect, in which an applied electric field produces a resulting strain in a body, is used to first convert an oscillating electric field applied to the left half of each bar into an extensional vibrational mode of the entire bar. If driven at resonance, standing-wave distributions of large amplitudes of stress and strain result. The resonantly amplified strain in the right half of each bar is then converted to a voltage across the output terminals by the direct piezoelectric effect. Depending upon the geometry and materials parameters, voltage amplifications of various magnitudes can be obtained, with associated step-downs in current levels.

Power Density Calculation

In designing piezoelectric transformers, it is useful to first investigate the fundamental limits of power density, the amount of power able to be transferred through the device per unit volume that a material can withstand. During each cycle of electrical input, some amount of work is transformed in the left half of the bar from electrical energy to mechanical energy and then similarly in the right half of the bar, that same mechanical energy is transformed back to electrical energy. The maximum power density could be limited by a number of mechanisms such as electric breakdown strength, maximum surface charge density, dynamic yield stresses or strains, dielectric or internal mechanical losses, or depoling due to electric fields or temperature rise. To address the power density issue, we consider a cellular element consisting of

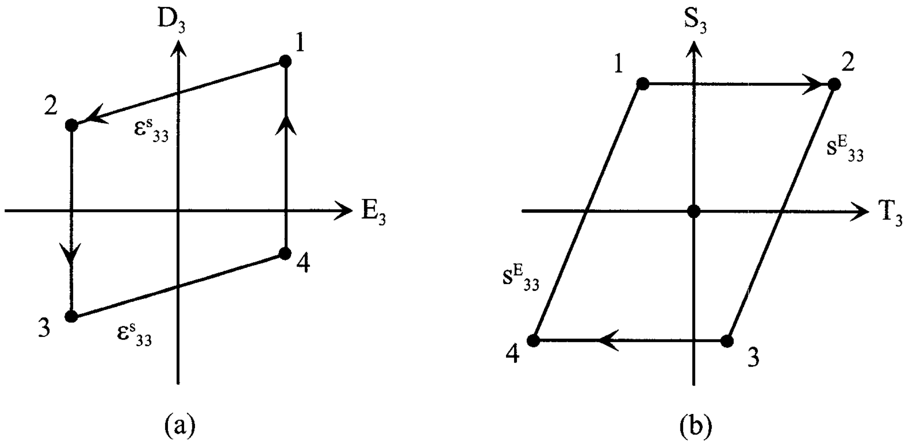

Figure 2. A work cycle viewed in both the electrical and mechanical domains. As the work cycle is traversed, the area inside each parallelogram is the energy transformed from electrical to mechanical energy or vice versa. (a) Electrical domain. (b) Mechanical domain.

one half of a piezoelectric transformer, such as the left half of the device in Figure 1(a).

We determine the power density by examining a work cycle. Let us start by assuming a lossless system that is in state 1 of its work cycle (point 1 of Figure 2). The structure is then blocked mechanically as the electric field is reduced first to $E_3=0$ and then to some negative value (point 2). At this point, the field is held constant and the mechanical block is removed, whereupon the structure moves from a state of positive strain to a state of negative strain (point 3), where the system has performed mechanical work. The element is then mechanically blocked again in this new strain state and the electric field is increased (point 4). The electric field is held constant and then the mechanical block is removed, moving the system back to its original state of positive strain (point 1), where mechanical work can be performed against a load. The cycle is repeated periodically and the power throughput density is equal to the energy density multiplied by the frequency of excitation, where the energy density is equal to the area of the parallelogram bounded by the trajectory of the work cycle. For idealized lossless operation, the areas of the two parallelograms are equal with one area corresponding to the electrical energy input and the other area corresponding to the mechanical energy output. Maximum power throughput depends on the maximum values of electric field, charge density, stress and strain which can be achieved with a given material. As we are only considering the lossless case here, in reality, the mechanical and dielectric dissipation must be taken into account to calculate more accurate figures of available power density.

We note that the trajectory of the work cycle of Figure 2 is actually consistent with a square-wave voltage drive, as readily achieved with a square-wave inverter, or some variant thereof. In this case, operation is at resonance, where the voltage drive is switched from maximum (point 1 of Figure 2) to minimum (point 2), when the strain, and corresponding displacement, is at its positive extreme. Conceptually, the voltage transition occurs instantaneously, and no change in the strain state occurs during this transition. The transition from point 3 to point 4 of Figure 2 occurs analogously, consistent with the symmetry of the square-wave drive.

Table I. PZT-5H Material Constants

K^T_3	Relative dielectric constant	3250	
$\varepsilon_{33}{}^T = K^T{}_3 * \varepsilon_0$	Dielectric constant at constant stress	2.88×10^{-8}	F/m
d_{33}	Piezoelectric strain coefficient	550×10^{-12}	m/V
$c_{33}{}^E$	Elastic modulus at constant E field	5.0×10^{10}	N/m^2
$s_{33}{}^E$	Elastic compliance at constant E field	2.0×10^{-11}	m^2/N
ρ	Density	7700	kg/m^3
E_{max}	Initial depolarization field	3.0×10^5	V/m
S_{max}	Maximum strain	0.002	
D_{max}	Maximum charge density	0.4	C/m^2
T_{max}	Maximum stress	1.4×10^7	N/m^2

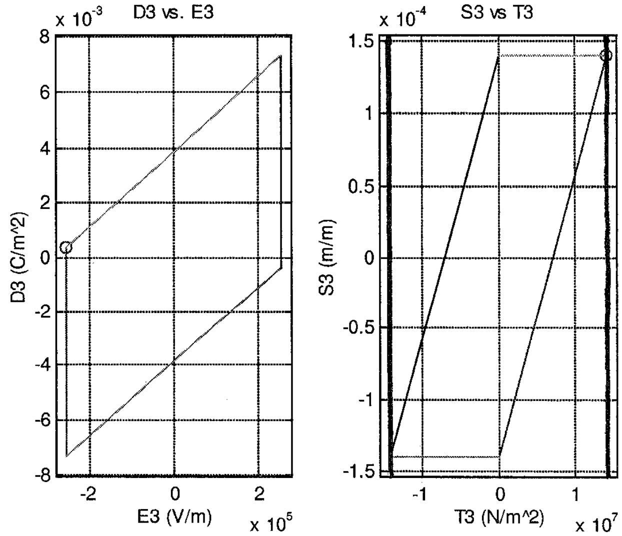

Figure 3. The work cycles for stress-limited PZT-5H yield an energy density of 3290 J/m^3.

The maximum-area work cycle will be bounded by one or more of the materials' limits, D_{max}, E_{max}, S_{max} or T_{max}. To explore the largest allowed work cycle, we pick an arbitrary corner such as (E_{max}, D_{max}) and map this state into the S-T domain. The piezoelectric constitutive equations [7] provide the mapping:

$$T_3 = (D_3 - \varepsilon_{33}{}^T E_3)/d_{33}$$

$$S_3 = s_{33}{}^E T_3 + d_{33} E_3$$

For an example material such as the PZT-5H ceramic [5], specified in Table I, the point (E_{max}, D_{max}) maps outside the T_{max}, S_{max} "bounding box". The material then is mechanical-field-strength limited. Assume the material is stress limited. From Figure 3(b), and known values of T_{max} and slope $s_{33}{}^E$, a value of strain can be found which maximizes the enclosed area. Figure 3 illustrates the work cycle for the example PZT-5H material with the characteristics of Table I. The enclosed

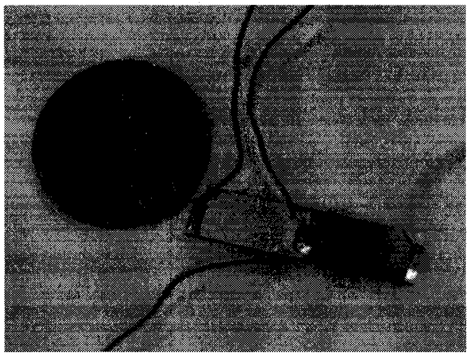

Figure 4. A prototype piezoelectric transformer was fabricated using PZT-4 material. The device is 24 mm x 6 mm x 0.5 mm.

areas are equal and give the maximum energy density in this stress-limited material. The calculated energy density is 3290 J/m^3. The resulting maximum power density, in the lossless case, at 100 kHz is thus 330 W/cm^3. For a ceramic piece 0.5 mm thick, this translates to 16.5 W/cm^2.

III Magnetic-Free Soft-Switched Drive

The preceding analysis was based on a square-wave drive, applied exactly at resonance. As reported in the literature [8-10], the impedance at the input terminals of the piezoelectric transformer is actually inductive at a frequency just above resonance, as is the case with a conventional series resonant L-C tank circuit. By operating the inverter at frequencies slightly above resonance, and by providing for a deadtime on the voltage transitions, it is possible to obtain resonant soft-switching. This constitutes a variation of the work cycle introduced in the preceding section. Specifically, in this case, the resonant mechanical motion of the transformer continues during the finite deadtime. A finite change in the stress state occurs during the deadtime, and so forces a corresponding change in charge-voltage state at the electrical terminals. The change in strain can easily be large enough to supply all the charge needed to drive the complete voltage transition during the deadtime, even considering the charge required by the parasitic capacitance of the small power MOSFETs used in the inverter.

Figure 5 illustrates a block diagram of a circuit realization of this inverter scheme. The power train is a half-bridge, always driven at 50% duty cycle. The gate drive block provides gate drive for the power train, along with appropriately adjusted dead times. The remainder of the circuit is a very simple feedback scheme to cause the inverter to frequency lock just above a resonant frequency. This is explained in the sequel.

Capacitors C_{S1} and C_{S2} form a capacitive voltage divider to allow sampling the large output voltage without overloading this very high impedance terminal. The phase shifter network, R_1-C_1 adjusts the phase of the output signal, which is then fed to a comparator to provide the inverter drive signal.

The use of an equivalent circuit model for the piezoelectric transformer, as shown in Figure 6, is most convenient for understanding the operation. In this model, the capacitor, $C_{compliance}$, represents the compliance, with state corresponding to strain. The inductor, L_m, represents the mass, with state corresponding to mechanical inertia. From inspection of the electrical circuit representation and corresponding waveforms, it is clear that a steady state oscillation above resonance is consistent with the description.

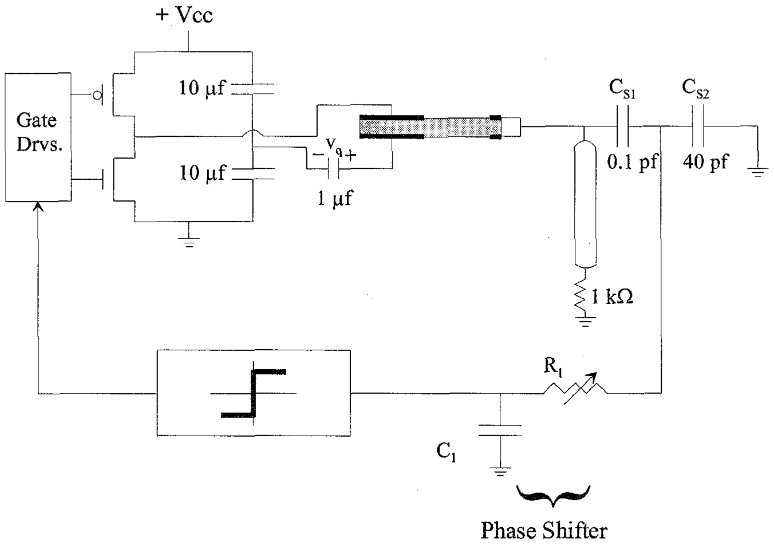

Figure 5. The primary of the piezoelectric transformer is driven by a MOSFET half-bridge and the cold cathode fluorescent lamp is the load on the secondary. A feedback signal is tapped off the transformer secondary via a capacitive voltage divider and fed back to the gate drivers to create a self-oscillating system. The feedback signal is phase shifted such that a zero-crossing comparator can produce the appropriate lock-on signal for the gate drivers.

An heuristic argument for stability can be motivated by the Nyquist criteria and the describing-function analysis method [6]. Basically, for a steady-state oscillation, a loop gain of unity and phase shift of some integral multiple of 2π is required. The phase condition is consistent with the indicated waveforms. The loop gain condition is achieved courtesy of the limiting behavior of the comparator non-linearity. Roughly, as the comparator input amplitude scales up, the effective gain of the loop is reduced. Thus, from a Nyquist stability point of view, if the amplitude is perturbed upward, the gain is reduced, making the system "more stable" and thus resulting in an amplitude reduction. An analogous argument can be made if the amplitude is perturbed downward.

IV Experimental Results

A prototype of the circuit of Figure 5 was constructed and used with a Rosen transformer of the type shown in Figure 1(b). The piezoelectric material was PZT-4, and overall dimensions were 48 mm x 8 mm x 2 mm. The second longitudinal extension mode resonance at 70 kHz yielded best results. A small cold cathode fluorescent lamp was used as the load. Due to the relatively thick PZT sample, a relatively high voltage of about 45 V was used to obtain a power output of about 500 mW. We note that thinner transformers, such as the prototype shown in Figure 4, can operate efficiently with lower input voltages.

For the circuit implementation of Figure 5, primary voltage and charge waveforms are shown in Figure 7(a). The waveform at top is the primary voltage applied by the inverter, at 20 V/div and 2 μs/div. Note the trapezoidal shape, indicative of the soft-switching action. The lower waveform in Figure 7(a) is representative of the charge on the primary, obtained by sampling across a 1 μF capacitor inserted in series with the primary. Scale for this waveform is 50 mV/div, or equivalently, 50 nC/div. Figure 7(b) provides a voltage-charge phase portrait for these same waveforms, by displaying charge on the vertical y-axis and voltage on the horizontal x-axis. Note the close resemblence to the electrical work cycle of Figure 2(a). The main

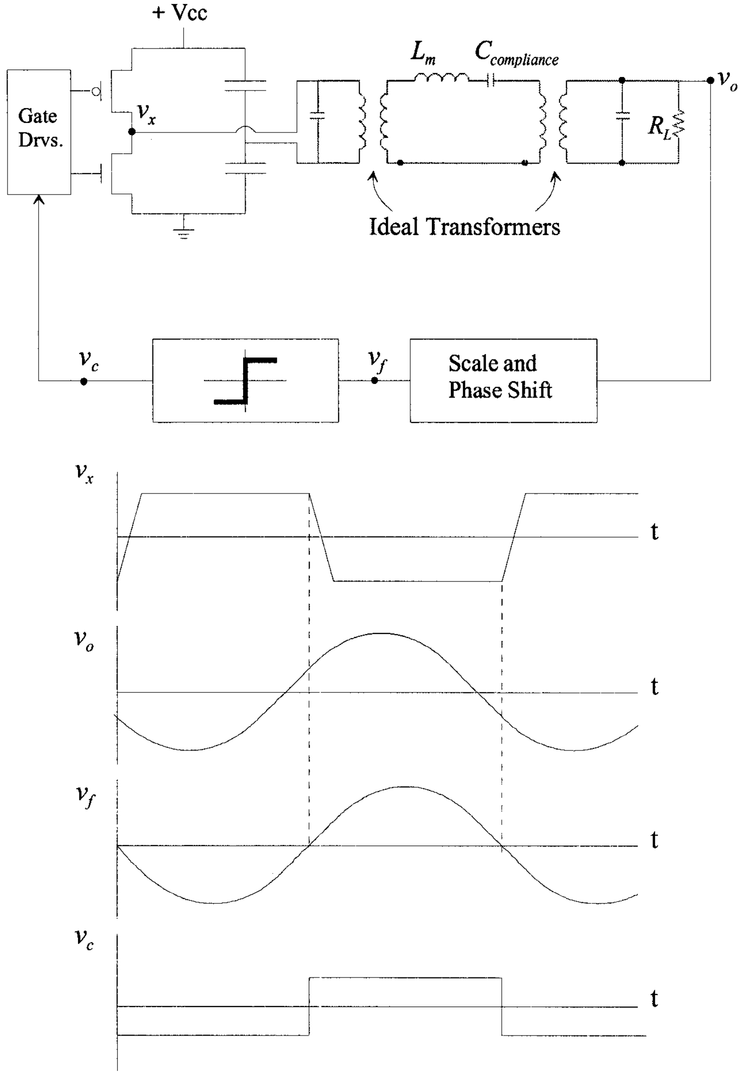

Figure 6. An equivalent model of the piezoelectric transformer consists of two ideal transformers which model the mechanical-electrical and electrical-mechanical energy conversion processes respectively. L_m and $C_{compliance}$ represent the resonant characteristic of the transformer. During the deadtimes when both MOSFETs are off and the drive frequency is just above resonance, the transformer acts inductive, creating a constant current source that pulls or pushes charge off the static input capacitance of the transformer and also the parasitic capacitances at the MOSFET inverter node. Since the MOSFETs are relieved from supplying the power to do this, switching losses are averted and efficiency is increased relative to a hard-switched configuration.

difference is that in the Figure 7(b), as the voltage increases, the charge actually *decreases*. The charge decrease is exactly the charge required to swing the parasitic capacitance associated with the MOSFET inverter output. By inspection of Figure 7, it is evident that this charge is about 15 nC, consistent with the charge needed to swing a parasitic output capacitance of about 330 pf through 45 V.

We note that if a hard-switched square-wave is applied at the primary, the switching loss is significant. A total capacitance of about 1 nF needs

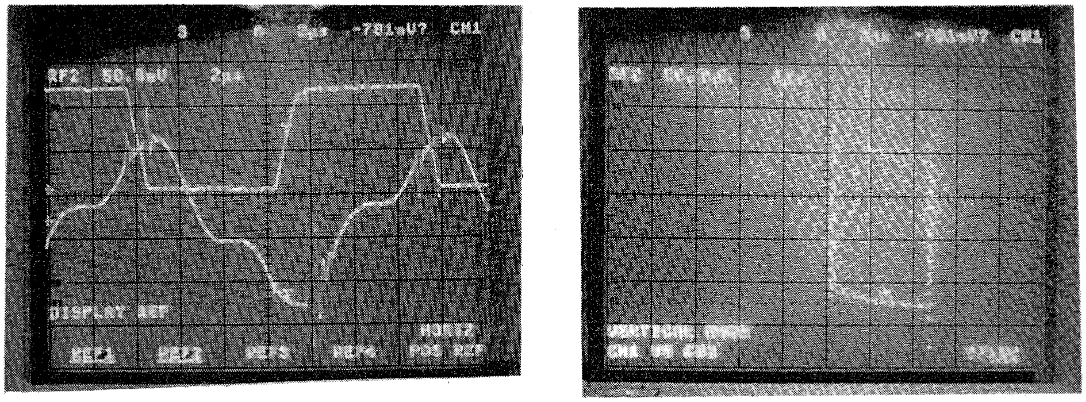

Figure 7. (a) The trapezoidal waveform at the top is the voltage applied to the transformer primary, labeled v_x in Figure 6. The vertical scale is 20 V/div and the horizontal scale is 2 µs/div. The lower trace represents the charge at that node, measured as the voltage (labeled v_q in Figure 5) across a 1 µf capacitor placed in series with the primary. The vertical scale of the lower trace is 50 mV/div, or equivalently, 50 nC/div. (b) Here, the two traces of (a) are plotted against each other with transformer primary charge, v_q, on the y-axis and primary voltage, v_x, on the x-axis, thus tracing out the electrical domain work cycle at the primary of the transformer. In comparing to Figure 2(a), note that hard-switching was assumed there, whereas soft-switching is achieved here. The change in charge during a low-to-high voltage transition is approximately 1/3 of a division *decrease.* That is, charge comes off the transformer primary to charge up the parasitic capacitance at the inverter node and swing the voltage up.

to be switched: about 300 pF associated with our inverter and approximately 700 pF associated with the static input capacitance of the transformer. The resulting CV^2f switching loss is then around 140 mW for a 45 V input voltage, and operation at 70 kHz. This comprises more than a 30% loss!

Figure 8 shows the secondary electrical waveforms along with the primary voltage (as in Figure 7(a)). The sinusoid-like waveform is a scaled version of the output voltage obtained through a 400:1 capacitive divider. This is shown on a 1 V/div scale, indicating an output voltage of about 1200 V peak. Note that the phase of this output voltage is related to the primary drive voltage, as shown in Figure 6, and previously discussed.

The third waveform represents the lamp current, sampled with a 1 kΩ resistor. This waveform is shown on a 1 V/div, or equivalently 1 mA/div, scale. Lamp current is thus about 2.4 mA peak. The phase shift between the lamp current and the transformer secondary output voltage is due to a small compensation capacitor of about 4 pF placed in series with the lamp. We note that accurate estimation of the output power from these waveforms is very difficult. In fact, estimating the angle between these waveforms at about 65 degrees, yields an output power of around 600 mW, which exceeds the measure DC input power of about 500 mW fed to the inverter! We conclude that an accurate efficiency measurement is difficult, but still believe the prototype system is quite efficient.

V Conclusion

Piezoelectric transformers offer comparable power densities and efficiencies to magnetic transformers, but are structurally simpler devices. The manufacturing process can also be much simpler as no windings or assembled cores are required. The thin, flat form factor of the piezoelectric transformer further enhances this packaging consideration. Significantly, we have shown that the entire driver for the cold cathode fluorescent lamp can be implemented without the need for any magnetic components.

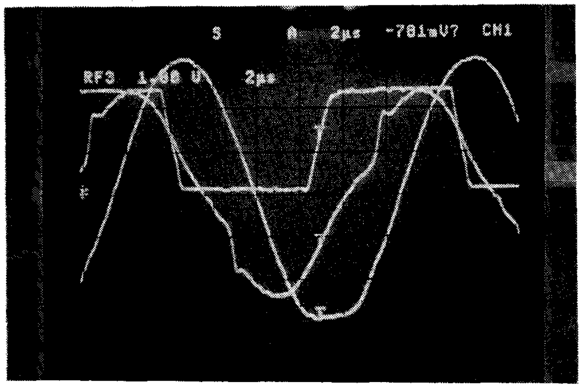

Figure 8. The trapezoidal waveform is again v_x, for reference, as in Figure 7. The smooth sine waveform is 1/400 of the voltage at the transformer secondary. That is, this waveform is the voltage fed into the feedback circuit from the capacitive voltage divider attached to the transformer secondary. The vertical scale of this feedback signal is 1 V/div. The horizontal axis is again 2 μs/div. The third waveform represents the current through the bulb, measure across a 1 kΩ resistor placed in series with the lamp. The vertical scale of the third waveform is 1 V/div, or equivalently, 1 mA/div.

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